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Modelling volatility spillovers between prices of petroleum and stock sector indices: A multivariate GARCH comparison

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Abstract: This study compares four multivariate GARCH approaches in modelling bilateral return and volatility spillovers between petroleum prices and self-constructed stock sector indices of net petroleum exporters (Canada and Saudi Arabia) and net petroleum importers (the United States and China). The estimates are subsequently used to quantify optimal portfolio weights and hedge ratios and to evaluate the effectiveness of the resulting hedging strategies. The outputs point to the presence of heterogeneous volatility interdependencies, which are more evident for Canada and the United States. The optimal weight of petroleum is greater in portfolios comprising stock sector indices of Saudi Arabia and China, which also provide lower hedging costs. Time-varying conditional correlations, portfolio weights, and hedge ratios exhibit considerable variations, particularly during turbulent periods. Finally, the hedging strategies generated from the VAR-DCC-GARCH specification result in the greatest reduction, although not substantial, of risks for portfolios involving stock sector indices of all countries.

Keywords: petroleum prices, stock sector returns, volatility transmission, multivariate GARCH, hedging effectiveness

JEL Classification: G11, G12, G15, Q43

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1. Introduction

Over the past fifteen years, the price of petroleum experienced extreme swings in its modern history, which coincided with analogous trends in stock markets. These periods of high volatility in both markets raise a question regarding contagion risks. A plethora of academic studies has emerged investigating the volatility transmission mechanisms between petroleum prices and stock markets (Malik & Hammoudeh, 2007; Malik & Ewing, 2009; Arouri et al., 2012; Mensi et al., 2013, among the notable works). Chang et al. (2010) define a volatility spillover as the lagged effects of variations in the volatility of one market on other markets' volatility. The strengthened integration of financial markets and the recent financialization of commodity markets have substantially enhanced the level of interconnection between petroleum and stock markets (Tang & Xiong, 2012; Silvennoinen & Thorp, 2013; Sadorsky, 2014a; Zhang et al., 2017). In this connection, accurate volatility modelling is of utmost appropriateness in modern finance, as good estimates of the dynamics of conditional variances and covariances are required for the efficient portfolio optimisation and management of risks (Sadorsky, 2012; Hamma et al., 2021).

To date, the multivariate GARCH models have been extensively utilised by many empirical studies to examine volatility dynamics between various commodities and equities. Some of these studies centre on stock markets from various regions, including the United States, Europe, as well as a group of petroleum-exporting and importing countries (Arouri et al., 2011a; Wang & Liu, 2016; Kartsonakis-Mademlis & Dritsakis, 2020; Wang et al., 2025). A number of studies concentrate exclusively on emerging markets (Sadorsky, 2014a; Hamma et al., 2014; Lin et al., 2014; Basher & Sadorsky, 2016; Hamma et al., 2021). Other studies consider clean energy, technology, and socially responsible stocks (Sadorsky, 2012; Sadorsky, 2014b). Several studies, albeit not directly related to the present work, analyse the performance of multivariate GARCH models in the context of different asset classes (Chang et al., 2011; Arouri et al., 2015; Zhong & Liu, 2021; Janda et al., 2022; Yadav et al., 2022; Guo & Zhao, 2024). One can draw several conclusions from the aforementioned studies. Some of them compare different GARCH techniques in volatility modelling and, based on the chosen specification, provide findings of optimal portfolio weights and hedge ratios. Others, which analyse the efficiency of hedge ratios across various GARCH models, consider specifications that do not always capture interactions in the conditional variance equations. In general, the DCC class models are reported to fit the data better and produce higher hedging effectiveness results. However, there are still areas to explore in understanding sector-specific dynamics, particularly in the context of petroleum exporters and importers.

Modelling the dynamics of volatility utilising multivariate GARCH approaches on a large dataset with multiple variables possesses challenges associated with the identification of a compromise between parsimony and flexibility when selecting appropriate specifications (Bauwens et al., 2006; Silvennoinen & Terasvirta, 2009; Basher & Sadorsky, 2016). Taking into consideration the existing literature, the present work aims to provide more insights into the performance of multivariate GARCH approaches in modelling volatility transmission mechanisms between petroleum and stock sector indices of petroleum-exporting and importing countries. In contrast to the previous studies, an investigation is conducted by comparing multivariate GARCH specifications that permit the measurement of interdependencies in both the conditional mean and variance equations between the studied variables. To this end, the current work focuses on stock sector indices of the top net petroleum exporters (Canada and Saudi Arabia) and importers (China and the United States), as portfolio managers tend to be interested in sector-level investment opportunities. It is worth emphasising that the aggregate-level of analysis is insufficient in obtaining a comprehensive picture, since sector weights across markets fluctuate based on their significance. Sectors exhibit varying degrees of reliance on petroleum, thus pointing to potential dissimilarities in their reactions to petroleum price swings. Conducting an investigation at the sector-level proves beneficial in this context, considering that it facilitates the identification of sector-specific effects. Given that the regulatory and construction methods vary among stock markets in the countries of interest, the study follows Bagirov & Mateus (2022) in order to manually build stock sector indices employing daily data on 1,658 individual stocks, which represent constituents of major aggregate market indices, and applying an equal-weighted approach. The innovative methodology of Bagirov & Mateus (2022) for the construction of sector indices enables a more detailed examination of cross-market spillover effects.

The study strives to accomplish three primary objectives: (i) to investigate return and volatility interdependencies between prices of Brent crude petroleum and six stock sector indices, namely Basic Materials, Consumer Cyclicals, Consumer Non-Cyclicals, Energy, Financials and Industrials, of petroleum exporting and importing countries by estimating VAR-BEKK-GARCH, VAR-GARCH, VAR-AGARCH and VAR-DCC-GARCH models; (ii) to derive, for portfolios composed of two assets, a stock sector index and petroleum, optimal wights and hedge ratios; and (iii) to analyse the performance of optimal hedge ratios derived from the considered four GARCH specifications in terms of risk minimisation utilising the hedging effective index. The study period runs from January 03, 2005, to September 28, 2018, which allows us to assess fluctuations in time-varying conditional correlations, portfolio weights, and hedge ratios during major global events.

The present study complements the growing body of literature on the dynamic interactions between petroleum prices and equity sectors by offering a methodologically robust analysis of volatility spillovers and hedging effectiveness through the use of multivariate GARCH models. In contrast to most of the existing literature, first, it systematically compares a range of multivariate GARCH specifications for each stock sector index-petroleum pairs, which explicitly capture interdependencies in both the conditional mean and conditional variance equations, thereby enhancing the understanding of risk transmission dynamics between petroleum prices and sectoral stock indices. It is worth mentioning that these models differ in both structure and parameterisation. VAR-BEKK-GARCH enforces positive definiteness of the conditional variance matrix automatically and allows for capturing intricate interactions between variables. VAR-GARCH employs a more streamlined structure, modelling conditional variances directly and providing consistent and asymptotically normal estimates under general conditions, although it does not account for asymmetries. VAR-AGARCH builds on the previous specification by incorporating asymmetric effects, allowing it to differentiate between the impacts of positive and negative shocks on volatility. VAR-DCC-GARCH separates the modelling of variances and correlations, allowing for time-varying conditional correlations between variables, making it more flexible in capturing changing market conditions. The advantage of such analysis is that the results provide a basis for an in-depth understanding of how hedging strategies differ by estimation technique. Second, it offers valuable insights for investors by concentrating on sector-level stock indices rather than aggregate markets, thereby identifying the heterogeneous sensitivities of sectors to petroleum price swings. The examination of dynamic conditional correlations, optimal portfolio weights, and hedge ratios derived from the multivariate GARCH estimates associated with each sector in petroleum-exporting and importing countries enables investors to customise hedging strategies at a granular level. Specifically, the study provides empirical evidence on how hedging effectiveness varies by sector and the country's net position in the global petroleum market, as well as by the underlying econometric model. These findings are of significant practical relevance, facilitating more efficient cross-asset portfolio construction, improved risk mitigation techniques, and more informed decision-making during turbulent market conditions. Third, the distinctive aspect is the focus on the key net petroleum exporters and importers, namely Canada, Saudi Arabia, the United States, and China. These four economies are not only among the most significant participants in global petroleum trade flows, but also exhibit differences in market maturity, regulatory environments, and energy policy approaches. This selection enables a comparative analysis that unravels the diverse effects of petroleum price volatility on equity sectors in exporting and importing countries, which enhances the understanding of the heterogeneity in petroleum-equity market volatility linkages.

The investigation unveils interesting outcomes. The estimates of multivariate GARCH models indicate the existence of shock or innovation and volatility interdependencies between stock sector indices and petroleum prices, which are more apparent for Canada and the United States. The course and intensity of observed volatility transmissions are contingent on sectors. It was detected that the VAR-AGARCH and VAR-DCC-GARCH specifications are superior in terms of fitting the dataset. The stock sector indices of Saudi Arabia and China exhibit the lowest time-varying conditional correlations with petroleum, despite the hikes over turbulent periods in the markets. Furthermore, the analysis of optimal portfolio holdings and hedge ratios indicates that, on average, irrespective of the models considered, the weight of petroleum is higher in portfolios encompassing stock sector indices of both countries, which also offer better possibilities for hedging exposure to petroleum price risks. The results of the VAR-DCC-GARCH model led to the highest variance reductions and, hence, the greatest hedging effectiveness, for portfolios involving stock sector indices of all countries, although it is worth mentioning that differences across models are not substantial. The dynamic portfolio weights and hedge ratios vary across sectors of net petroleum exporters and importers, and display significant sensitivities to major events that caused instabilities in stock and petroleum markets.

The remainder of this study is organised as follows. Section 2 reviews the existing literature. Section 3 describes data, construction processes of stock sector indices and descriptive statistics. Section 4 presents four multivariate GARCH specifications. Section 5 discusses the empirical findings, including constant and dynamic conditional correlations, from the estimated models. Section 6 analyses optimal portfolio holdings and hedge ratios, along with the effectiveness of hedging strategies. Section 7 outlines implications. Section 8 finalises the study.

2. Literature review

The present work confines the review of the literature to the relevant papers that compare and contrast the performance of different multivariate GARCH approaches in modelling volatility spillovers between petroleum and stock markets, and analyse the effectiveness of optimal hedge ratios. Several studies examine stock markets from multiple countries. Arouri et al. (2011a) use four multivariate GARCH models, such as VAR-GARCH, CCC-GARCH, diagonal BEKK-GARCH, and DCC-GARCH, to investigate volatility transmissions between petroleum prices and stock sectors in the United States and Europe over the period from 1998 to 2009. The findings related to the VAR-GARCH model, which permits capturing interactions, indicate that the volatility spillovers are generally unilateral from petroleum prices to stock sectors in Europe, and bilateral between petroleum prices and stock sectors in the United States. The portfolio weights and hedge ratios computed from GARCH models do not display substantial variability, but differ across sectors of the two markets. The authors provide evidence of the superiority of hedged portfolios over traditional portfolios composed of stocks only, irrespective of the models. In addition, the VAR-GARCH model performs better in terms of hedging effectiveness. Wang & Liu (2016) consider seven petroleum exporters and nine petroleum importers to study volatility interactions between petroleum and stock markets. The authors employ different GARCH class models and weekly data for the period 2000-2011. Their results obtained from the BEKK-GARCH approach point to the presence of volatility spillovers, which are contingent on the countries' level of petroleum exports and imports. Furthermore, the strategies based on the DCC-GARCH and RS-DCC approaches produce the highest hedging effectiveness in most cases. Kartsonakis-Mademlis & Dritsakis (2020) analyse the transmission of volatility between Brent crude petroleum prices and the stock markets of the G7 countries using various asymmetric multivariate GARCH models. Their findings reveal that the choice of model impacts the identification of spillover effects, with the asymmetric BEKK model outperforming alternatives. Additionally, the study demonstrates that including petroleum in equity portfolios enhances hedging effectiveness. Wang et al. (2025) investigate interactions between crude petroleum ETFs, renewable energy ETFs, and the S&P 500 Index ETF utilising three multivariate GARCH specifications, such as BEKK-GARCH, CCC-VARMA-GARCH, and DCC-VARMA-GARCH. Their results indicate that volatility spillovers from equities to both traditional and renewable energies are positive in the short term but become negative over longer horizons. The authors also found that the DCC-VARMA-GARCH-in-mean model demonstrates the best overall performance.

A group of works focuses on emerging markets. Sadorsky (2014a) applies CCC-AGARCH, VARMA-AGARCH, and DCC-AGARCH models to analyze volatility linkages and time-varying correlations between prices of the MSCI Emerging Markets Index and four types of commodities, namely petroleum, copper, and wheat, over the period from January 2000 to June 2012. The VARMA-AGARCH estimates provide evidence of long-term volatility spillovers between the emerging stock market index and petroleum. However, the residual diagnostics tests, AIC, and SIC criteria suggest that the DCC specification fits the data better, which is subsequently used to construct dynamic correlations, portfolio weights, and hedge ratios. Furthermore, the time-varying hedge ratios and conditional correlations between the considered asset classes increased substantially during the recession. Hamma et al. (2014) examine volatility spillovers between petroleum and the Tunisian aggregate market index, including seven sector indices, and the effectiveness of hedging strategies. The authors utilise three multivariate GARCH models, such as BEKK-GARCH, CCC-GARCH, and DVEC-GARCH, and weekly data covering the period from April 2006 to July 2012. The results associated with the BEKK-GARCH model indicate that volatility transmissions are mostly unidirectional from the petroleum market to stock sector indices. The optimal portfolio weights and hedge ratios vary across models and sectors. In addition, the authors found that the BEKK-GARCH model is more efficient in minimizing risks of petroleum-stock sector portfolios. Lin et al. (2014) estimate that DCC-GARCH, VAR-GARCH, and VAR-AGARCH models employ weekly data over the period from 2000 to 2010 to study volatility spillovers between petroleum and stock markets in Ghana and Nigeria. All models provide evidence of volatility interdependencies between petroleum and both stock markets. The authors, while not using the traditional hedging effectiveness index, show that the DCC-GARCH framework results in more effective hedging. Basher & Sadorsky (2016) investigate the performance of three multivariate GARCH approaches, namely DCC, ADCC, and GO-GARCH, in modelling volatility dynamics and optimal hedge ratios between the emerging stock market, represented by the MSCI Emerging Markets Index, and petroleum, gold, bonds, and VIX for the period from January 2000 to July 2014. Their findings indicate that the best fitting specification is ADCC, which produces the highest hedging effectiveness between the emerging stock market and petroleum, bonds, and VIX. In the majority of cases, petroleum is detected to be the more suitable asset in terms of hedging prices of emerging market stocks. However, such a conclusion cannot be applied to all emerging countries due to aggregation in the index's construction process. Hamma et al. (2021) compare three competing DCC specifications on hedging Islamic and emerging stock markets with different types of financial assets, such as crude petroleum, gold, CDS Europe Index, VIX, EURO STOXX 50 Volatility Index, and Dow Jones Commodity Index. Specifically, the authors estimate DCC, ADCC, and FDCC models utilising daily data from December 2007 to September 2016. The results suggest that hedged portfolios composed of mixed assets outperform conventional portfolios of stocks. Furthermore, the DCC model mostly leads to a greater reduction of hedged portfolios' variances than the other models used.

Other studies examine volatility spillovers by focusing on asset groups with distinct technological and environmental profiles. Sadorsky (2012) examines volatility transmissions between prices of petroleum and stocks of technology and clean energy companies over the period from 2001 to 2010. The author employs four multivariate GARCH models that capture volatility interdependencies, namely BEKK, Diagonal, CCC, and DCC, where the conditional variance equations of the latter three follow the VARMA-GARCH specification of Ling & McAleer (2003). The DCC model is found to be the best, although the BEKK model provides stronger evidence of volatility interactions. The estimates of the DCC model show the existence of shock and volatility spillovers from petroleum to clean energy stocks. Sadorsky (2014b) uses DIAG-GARCH, CCC-GARCH, and DCC-GARCH models, which do not incorporate cross effects in the conditional variance equations, and weekly data covering the period from December 1998 to May 2012 model volatility and dynamic correlations between stocks of the socially responsible firms, represented by the Dow Jones Sustainability Index (DJSI), S&P 500 Index, petroleum, and gold. The DCC-GARCH model is detected to fit the data better among the three approaches. The results indicate that dynamic correlations of the DJSI and S&P 500 indices with petroleum exhibit analogous patterns. In addition, the figures of portfolio weights and hedge ratios between these assets are similar, suggesting that investors considering socially responsible and S&P 500 stocks can expect similar costs in order to hedge their positions with petroleum.

Some prior studies consider various asset classes without direct emphasis on petroleum-stock volatility transmission dynamics. For example, Chang et al. (2011) examine the effectiveness of hedging strategies between spot and futures returns of two major crude petroleum benchmarks, namely Brent and WTI, employing BEKK, Diagonal BEKK, CCC, DCC, and VARMA-GARCH models. The findings show that optimal hedge ratios derived from the Diagonal BEKK and DCC models lead to greater and from the BEKK model to lesser reduction of risks. Arouri et al. (2015), utilising six models, such as BEKK-GARCH, Diagonal BEKK-GARCH, Scalar BEKK-GARCH, CCC-GARCH, DCC-GARCH, and VAR-GARCH, investigate volatility transmissions between global gold prices and the Chinese stock market. The authors document the superiority of the VAR-GARCH specification, which also results in better diversification and hedging effectiveness. Guo & Zhao (2024) analyse volatility transmissions between crude petroleum and coal prices in China using daily data and four different multivariate GARCH models. The study identifies the DCC-GARCH model as the most fitting for the employed dataset. Yadav et al. (2022) study volatility transmissions and correlations between stock markets of China and selected Asian and Latin American countries, focusing on the VARMA-MGARCH framework with BEKK, diagonal, CCC, and DCC specifications. The authors detected that the DCC model outperforms the other four examined specifications.

The literature underscores the importance of selecting appropriate multivariate GARCH models for analysing cross-market volatility transmission, particularly within the context of petroleum and stock markets. It is generally observed that the DCC class models provide a better fit for the data and yield superior results in terms of hedging effectiveness.

3. Data and descriptive statistics

To examine the variability of volatility interdependencies between prices of petroleum and stock sector indices of net petroleum exporters and importers, the study chooses countries with developed and emerging markets that are the largest producers and consumers of petroleum based on data provided by the British Petroleum Statistical Review of World Energy. The sample comprises two petroleum-exporting countries, Canada and Saudi Arabia, and two petroleum-importing countries, China and the United States. In 2018, Canada and Saudi Arabia produced approximately 5.208 and 12.287 million barrels per day, respectively (BP Statistical Review of World Energy 2019). The level of petroleum production exceeded petroleum consumption during the period of analysis in both countries. China and the United States consumed around 13.525 and 20.456 million barrels per day in 2018, respectively (BP Statistical Review of World Energy 2019). The countries' level of petroleum consumption surpassed petroleum production during the entire investigation period.

The Brent grade of crude petroleum is used to represent the petroleum market, which is one of the main market grades and serves as a reference price for nearly two-thirds of global crude petroleum spot trades. The study of Batten et al. (2021) emphasises the suitability of the Brent grade for the stock/petroleum hedging analysis, as on average, it offers greater hedging effectiveness compared to the WTI grade, the other widespread benchmark. The spot prices of Brent crude petroleum expressed in US dollars were obtained from the Datastream database, where the data provider is the US Energy Information Administration.

The study utilises daily data covering the period from January 03, 2005, to September 28, 2018. The employment of such frequency is more appropriate for investigating volatility spillovers, as important short-term information inherent in daily data is not lost (Mensi et al., 2015; Maghyereh et al., 2017). The data range is motivated by the objective of capturing the effects of major global events that caused significant instabilities in the stock and petroleum markets, thereby offering valuable insights into volatility interactions. Taking into account the fact that pricing and trading of the Brent grade takes place in the US dollar, it is selected as the main currency.

Given that the stock markets in the countries of interest follow their own regulatory and construction processes, the study adopts a novel methodology of Bagirov & Mateus (2022) in order to manually construct stock sector indices employing data on individual stocks, which permits the application of the same approach across all markets. To select the list of sectors and individual stocks, the following major stock market indices were used: the S&P TSX Composite index for Canada, the Tadawul All Share index for Saudi Arabia, the CSI 300 index for China, and the S&P 500 index for the United States. The considered aggregate market indices follow different industry classification standards. For instance, the S&P TSX Composite, Tadawul All Share, and S&P 500 indices adopted GICS, and the CSI 300 index adopted the CSI Industry Classification Standards (The Tadawul All Share index followed the local sector classification scheme until January 2017). Thus, the study opts for the Thomson Reuters Business Classification (TRBC) to standardise the classification schemes. Afterwards, the quarterly constituent lists of aggregate market indices were obtained from the Datastream database, where the main criterion was that sectors should have consisted of at least five stocks in each quarter over the entire period of investigation to be eligible for selection. It is worth noting that in the case of Saudi Arabia, the number of stocks listed in the Consumer Cyclicals and Consumer Non-Cyclicals sectors until the 3rd quarter of 2005 and the Energy sector until the 1st quarter of 2006 were below the set criterion. Considering the importance of this largest petroleum exporter, it was decided to construct sector indices using available stocks to ensure that the starting period remains the same in all four markets.) The selection process resulted in 1,658 unique stocks from six sectors, such as Basic Materials, Consumer Cyclicals, Consumer Non-Cyclicals, Energy, Financials, and Industrials, of Canada, China, Saudi Arabia, and the United States, which are reported in Table 1 (the adopted selection criteria ensure sufficient representation of each sector, enabling meaningful analysis at the sector level rather than based on a limited set of individual stocks and facilitating more robust comparisons across countries).. The sectors that constitute the greatest proportion of the total sample are Basic Materials and Financials, where their share is more than 40 percent. At the individual market level, the Financials and Industrial sectors dominate in China and the United States, while the Basic Materials and Energy sectors are more important in Canada, and the Basic Materials and Financials sectors stand out with the largest number of stocks in Saudi Arabia.

Table 1. Total number of stocks listed in the six sectors of petroleum exporters and importers.

Country/Index	Basic Materials	Consumer Cyclicals	Consumer Non-Cyclicals	Energy	Financials	Industrials
Panel A: Next exporters						
Canada/S&P TSX	151	44	20	129	74	42
Saudi Arabia/TADAWUL	42	18	20	6	67	22
Panel B: Net importers						
China/CSI 300	85	82	38	37	98	124
United States/S&P 500	48	138	63	65	145	100
Grand Total	326	282	141	237	384	288

The adjusted closing prices were retrieved from the Datastream database and converted from local currencies to US dollars. Following completion of the sampling processes, the study employs an equal-weighted approach to manually build sector indices for all markets as follows (It is generally considered to have a lesser inclination towards favouring stocks with high prices and large market capitalization):

$$I_{i,t} = I_{i,t-1} \times (1 + r_{i,t}). \quad (1)$$

where the base level of sector indices commences from 100, the value of the ith sector index at time t is represented by $I_{i,t}$, $I_{i,t-1}$ signifies the value of the ith sector index at time t-1, and $r_{i,t}$Denotes the average of daily logarithmic returns of all individual stocks listed in the ith sector at time t. To consider joining and leaving firms and, hence, to maintain the continuity of indices, during the construction process, all stock sector indices were readjusted on a quarterly basis.

The main descriptive statistics of daily returns for Brent crude petroleum and six sector indices of petroleum-exporting and importing countries are displayed in Tables 2 and 3, respectively. The daily returns of stock sector indices and crude petroleum prices are computed as $r_{i,t} = ln(P_{i,t}/P_{i,t-1}) \times 100$. The highest average daily returns were produced by Brent crude petroleum, the Consumer Non-Cyclicals sector indices of Canada and China, and the Industrial sector index of the United States. Conversely, the Energy sector index of Canada and the Industrial sector index of Saudi Arabia generated the lowest average daily returns. Additionally, it can be observed that, on average, more sector indices of petroleum-exporting countries compared to their counterparts in the petroleum-importing countries experienced negative returns during the sample period. The highest maximum daily returns were exhibited by the Energy stock sector indices in the case of Canada, Saudi Arabia, and the United States, and by the Industrials sector index in the case of China. The Energy sector indices of Canada and the United States, the Consumer Cyclicals sector index of China, and the Consumer Non-Cyclicals sector index of Saudi Arabia showed the lowest minimum daily returns. Brent crude petroleum had relatively greater daily maximum and lower daily minimum returns than sector indices of exporters and importers, which is not surprising in light of the remarkable petroleum price swings over the past decades. With regard to the standard deviation figures, the Basic Materials sector index of Canada, the Consumer Non-Cyclicals sector index of Saudi Arabia, the Energy sector indices of China and the United States, and Brent crude petroleum are riskier. The daily returns for all stock sector indices have negative skewness statistics, except for Brent crude petroleum, suggesting that the series have long left tails. Kurtosis statistics exceed the value of 3 for all return series, apart from the Energy sector index of China, signifying the existence of fat tails. The normality condition is rejected at the significance level of 1% for each of the return series as indicated by the Jarque-Bera test. The statistically significant values of the ARCH effects in the case of all series signify the appropriateness of multivariate GARCH models. Applying the Augmented Dickey-Fuller test, where the number of lags was chosen based on the Schwarz Bayesian Criterion, it was detected that the null hypothesis of the presence of a unit root is rejected at the 1% significance level in all cases, which suggests that the series are stationar (when the test was carried out using the Akaike Information Criterion to select the number of lags, the series are also found to be stationary). The unconditional correlations between stock sector indices and daily returns of Brent crude petroleum indicate that values are positive and substantially vary across sectors of petroleum-exporting and importing countries. Expectedly, Brent crude petroleum and Energy sectors of Canada and the United States have the highest correlations, 0.5392 and 0.4172, respectively, while the lowest correlation of 0.0809 is observed for the Consumer Non-Cyclicals sector of China. It is interesting to note that the stock sector indices of Saudi Arabia and China are weakly correlated with Brent crude petroleum. On the contrary, the correlation levels are greater between Brent crude petroleum and the stock sector indices of Canada and the United States.

Table 2. Descriptive statistics of daily returns for Brent and stock sector indices of petroleum-exporting countries.

Country/Index	Mean (%)	Max (%)	Min (%)	Std. dev. (%)	Skewness	Kurtosis	Jarque-Bera	ARCH	ADF	Corr. with Brent
Panel A: Canada										
Brent	0.0208	18.1297	-16.832	2.1316	0.1004	5.1219	3778.04***	214.82***	-57.92***	1
Basic Materials	-0.035	14.6063	-16.7692	2.2816	-0.6882	5.9034	5283.48***	730.32***	-55.65***	0.3697
Consumer Cyclicals	-0.0012	7.9344	-6.7825	1.183	-0.3678	3.4276	1767.19***	534.13***	-53.46***	0.3754
Consumer Non-Cyclicals	0.0223	6.6234	-8.5685	1.0367	-0.4513	5.8134	4976.64***	436.98***	-56.67***	0.3385
Energy	-0.0433	14.6966	-17.9464	2.0745	-0.6518	6.799	6891.36***	775.20***	-53.80***	0.5392
Financials	0.0036	9.9836	-14.4381	1.2833	-0.6243	12.4966	22679.4***	622.40***	-27.83***	0.402
Industrials	0.0045	10.1419	-11.1191	1.3219	-0.7023	8.1828	9911.81***	744.00***	-53.39***	0.4005
Panel B: Saudi Arabia										
Brent	0.021	18.1297	-17.4698	2.1315	0.0391	5.4371	4215.95***	169.02***	-57.08***	1
Basic Materials	-0.0241	9.021	-17.1868	1.9318	-1.4101	9.5885	14243.1***	822.32***	-25.5***	0.1765
Consumer Cyclicals	-0.0284	11.0519	-13.5887	2.0917	-1.1113	7.5636	8861.34***	911.00***	-24.86***	0.1193
Consumer Non-Cyclicals	-0.0164	12.3461	-18.9908	2.3871	-1.0712	6.9321	7506.1***	969.88***	-23.56***	0.101
Energy	-0.0193	13.3606	-17.6706	2.2477	-1.0452	7.6266	8916.39***	909.77***	-24.92***	0.1384
Financials	-0.0355	9.2216	-11.8997	1.8384	-1.2887	6.6625	7276.17***	572.10***	-30.66***	0.146
Industrials	-0.0498	12.4487	-15.8752	2.2684	-1.1973	6.8805	7567.62***	950.40***	-24.45***	0.1344

Notes: ADF denotes the Augmented Dickey-Fuller unit root test, where the critical value at the 1% significance level is -3.4352. ARCH refers to Engle's test for autoregressive conditional heteroscedasticity of order five, which is applied as the preliminary test for the existence of ARCH effects in residuals. *** indicates the significance level at 1%.

Table 3. Descriptive statistics of daily returns for Brent and stock sector indices of petroleum-importing countries.

Country/Index	Mean (%)	Max (%)	Min (%)	Std. dev. (%)	Skewness	Kurtosis	Jarque-Bera	ARCH	ADF	Corr. with Brent
Panel A: China										
Brent	0.0215	24.6651	-17.4698	2.1724	0.2372	9.2992	12072.9***	77.52***	-56.45***	1
Basic Materials	0.0011	8.9336	-11.1598	2.2235	-0.7325	3.225	1747.16***	412.23***	-54.15***	0.1163
Consumer Cyclicals	0.0185	8.7854	-11.3154	2.0245	-0.7976	3.7305	2292.25***	350.56***	-53.95***	0.0898
Consumer Non-Cyclicals	0.0409	8.8881	-10.7711	1.9422	-0.7137	3.7757	2268.77***	405.2***	-53.95***	0.0809
Energy	-0.0114	8.7169	-11.265	2.2822	-0.4942	2.7207	1166.8***	313.42***	-54.68***	0.1406
Financials	0.0157	8.8638	-10.9588	2.0787	-0.5447	3.0688	1476.62***	235.14***	-55.87***	0.0861
Industrials	0.0065	9.1146	-10.8336	2.0008	-0.762	3.8956	2436.58***	458.43***	-53.85***	0.0916
Panel B: United States										
Brent	0.0207	18.1297	-16.832	2.1262	0.1042	5.0945	3746.83***	203.41***	-57.81***	1
Basic Materials	0.0028	10.9197	-13.8826	1.5763	-0.7125	7.9892	9491.7***	886.10***	-60.65***	0.2631
Consumer Cyclicals	0.0059	10.5627	-10.636	1.4691	-0.4085	8.2406	9883.28***	1013.38***	-57.80***	0.1752
Consumer Non-Cyclicals	0.0193	8.394	-7.2547	0.9099	-0.4754	9.5857	13373.4***	956.77***	-46.86***	0.1316
Energy	-0.0111	18.1411	-20.5845	2.0936	-0.955	11.5795	19850.8***	849.97***	-45.29***	0.4172
Financials	-0.0173	14.2906	-19.709	1.9584	-0.9218	17.41	44175.4***	677.99***	-66.58***	0.1432
Industrials	0.0195	9.4564	-10.9082	1.3412	-0.5573	7.6622	8640.6***	857.96***	-60.98***	0.2154

Notes: ADF denotes the Augmented Dickey-Fuller unit root test, where the critical value at 1% significance level is -3.4352. ARCH refers to the Engle's test for autoregressive conditional heteroscedasticity of order five, which is applied as the preliminary test for the existence of ARCH effects in residuals. *** indicates the significance level at 1%.

4. Multivariate GARCH models

Effective volatility modelling and comprehending interdependencies between financial variables are crucial for optimising investment portfolios and hedging risks. The multivariate GARCH models have been documented as advantageous in this regard (Although we acknowledge the recent advancements in volatility transmission methods based on the time-varying connectedness approaches, the choice was made in favour of multivariate GARCH models considering the objectives of the present paper that require conditional variance and covariance estimates.). It is therefore essential to select appropriate specifications from a broad spectrum of models. The present work investigates the performance of four multivariate GARCH approaches in modelling volatility transmissions between petroleum prices and manually built stock sector indices of the largest petroleum exporting and importing countries, which contain two elements: (i) a conditional mean equation specified as the vector autoregressive (VAR) process; and (ii) a conditional variance equation specified as four multivariate GARCH processes. The considered specifications of models allow for capturing volatility cross-effects between the studied variables.

The conditional mean equation is modelled utilising the VAR process with one lag for each pair involving the stock sector index and petroleum returns, which is structured as follows (The optimal lag length based on the Schwarz Bayesian Criterion is one in most cases (Appendix, Table A.1):

$$R_{i,t} = \mu_i + \sum_{j=1}^{n} \phi_{ij} R_{i,t-1} + \varepsilon_{i,t}. \tag{2}$$

$$\varepsilon_{i,t} = h_{i,t}^{1/2} \eta_{i,t}$$

where, μ_i is the $n \times 1$ vector of constant terms, $R_{i,t}$ denotes the vector of daily logarithmic returns of the ith stock sector index and Brent crude petroleum prices at time t, ϕ_{ij} is the $n \times n$ matrix of coefficients that permits the cross-dependency between two series in the conditional mean equation, $\varepsilon_{i,t}$ refers to the $n \times 1$ vector of error terms, $\eta_{i,t}$ is the $n \times 1$ vector of i.i.d. random errors, and $h_{i,t}$ signifies the conditional variance of the ith stock sector index and Brent crude petroleum price returns at time t.

The first considered approach is BEKK proposed by Engle & Kroner (1995). The conditional variance equation of BEKK (hereinafter referred to as the VAR-BEKK-GARCH model) for the multivariate GARCH(1,1) process is defined as follows:

$$H_t = CC' + A'^{\varepsilon_{t-1}\varepsilon'_{t-1}} A + B'^{H_{t-1}} B. \tag{3}$$

where, C is the $n \times n$ lower triangular matrix of constant terms with elements c_{ij}, A and B are the $n \times n$ matrices of coefficients with elements α_{ij} and β_{ij}, which represent ARCH and GARCH terms, respectively, and measure the impact of past own shocks or innovations and volatilities, including the spillover effects between the ith stock sector index and petroleum. The attractive property of this model is that the positive definiteness of H_t is enforced automatically. However, because of several transpositions of matrices, the model's estimation bears heavy computations (Silvennoinen & Terasvirta, 2009). Furthermore, a large number of parameters, including additional variables, causes difficulties in obtaining convergence.

The conditional variance equation of the second considered approach follows the GARCH(1,1) process (hereinafter referred to as the VAR-GARCH model) with the specification of Ling & McAleer (2003) (Chang et al. (2011) suggest that practically multivariate GARCH models with the longer number of p and q lag orders can cause difficulties during the estimation process.):

$$H_t = C + \sum_{j=1}^{n} A_j \varepsilon_{t-1}^2 + \sum_{j=1}^{n} B_j H_{t-1}. \tag{4}$$

where, C denotes the $n \times 1$ vector of constant terms, A_j and B_j refer to the $n \times n$ matrices with elements α_{ij} and β_{ij}. For $i = j$, α_{ij} represents the ARCH term that measures the effects of past own innovations or shocks, and β_{ij} represents the GARCH term that measures the effects of past own volatilities. For $i \neq j$, this approach permits past shocks and volatilities of one variable to impact volatilities of other variables in the system. Specifically, the coefficients α_{ij} and β_{ij} indicate that the conditional variance of the ith stock sector index (or Brent crude petroleum prices) is affected by past squared errors and variances of Brent crude petroleum prices (or the ith stock sector index), respectively. Although the VAR-GARCH model captures the spillover effects, it assumes that positive and negative shocks of the same magnitude have an identical impact on the conditional variance.

The work of McAleer et al. (2009) extends the aforementioned specification in order to consider the asymmetric influences of shocks, both positive and negative. The conditional variance equation of the third approach (hereinafter referred to as the VAR-AGARCH model) is specified as follows:

$$H_t = C + \sum_{j=1}^{n} A_j\, \varepsilon_{t-1}^2 + \sum_{j=1}^{n} D_j\, I_{t-1}\varepsilon_{t-1}^2 + \sum_{j=1}^{n} B_j\, H_{t-1} \tag{5}$$

where, D_j represents the $n \times n$ matrices, and $I_t = diag(I_{1t}, \dots, I_{nt})$ refers to the indicator variable that separates between the positive and negative effects of shocks of the same magnitude on the conditional variance, and equals one if $\varepsilon_{it} \leq 0$ and zero if $\varepsilon_{it} > 0$. The positive value of D_j This specification implies that negative residuals have a tendency to increase the conditional variance more compared to positive residuals. Both the VAR-GARCH and VAR-AGARCH models embody the constant conditional correlation (CCC) GARCH process of Bollerslev (1990). The conditional covariance between the ith stock sector index and Brent crude petroleum is defined as follows:

$$h_{ij,t} = \rho_{ij} h_{ii,t}^{1/2}\, h_{jj,t}^{1/2}. \tag{6}$$

where the constant conditional correlation is represented by ρ_{ij}. The works of Ling & McAleer (2003) and McAleer et al. (2009) have documented in detail the necessary and sufficient conditions for the VAR-GARCH and VAR-AGARCH models.

The assumption of correlations being time-invariant can be viewed as impractical, given that correlations tend to change over time in accordance with market conditions. Engle (2002) proposed the Dynamic Conditional Correlation (DCC) model in order to allow conditional correlations between the studied series to be time-varying, which is expressed as follows:

$$H_t = D_t R_t D_t. \tag{7}$$

where, H_t denotes the conditional covariance matrix, $D_t = diag(h_{11,t}^{1/2}, \dots, h_{nn,t}^{1/2})$ refers to the diagonal $n \times n$ matrices of conditional variances and R_t represents the $n \times n$ conditional correlation matrix. The fourth approach (hereinafter referred to as the VAR-DCC-GARCH model) is estimated using the two-step procedure. First, the multivariate GARCH estimates are obtained from the GARCH(1,1) process that follows the specification of Ling & McAleer (2003) expressed in the third equation. The recent empirical studies that have applied such specification within the DCC framework to study time-varying volatility and correlation dynamics between different markets include Sadorsky (2012), Ahmad (2017), Maghyereh et al. (2017), and Bagirov & Mateus (2022). Second, the time-varying conditional correlation matrix is estimated as follows:

$$R_t = diag(q_{11,t}^{-1/2}, \dots, q_{nn,t}^{-1/2}) Q_t diag(q_{11,t}^{-1/2}, \dots, q_{nn,t}^{-1/2}). \tag{8}$$

Q_t, which denotes the $n \times n$ symmetric positive definite matrix, is given as:

$$Q_t = (1 - \theta_1 - \theta_2)\overline{Q} + \theta_1 \varepsilon_{t-1} \varepsilon'_{t-1} + \theta_2 Q_{t-1}. \tag{9}$$

where, θ_1 and θ_2 refer to scalar parameters that are non-negative, where $\theta_1 + \theta_2 < 1$, and capture the past effects on the current conditional covariances, and $\overline{Q}$ represents the $n \times n$ matrix of the unconditional covariance of standardised residuals ε_t. The dynamic conditional correlation estimator between the ith stock sector index and Brent crude petroleum is defined as:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}. \tag{10}$$

Given that for financial time series the normality condition is not often met, the quasi-maximum likelihood estimation (QMLE) method is applied to estimate the VAR-BEKK, VAR-GARCH, VAR-AGARCH and VAR-DCC-GARCH approaches. The conditional variance and covariance estimates are then utilised to construct optimal portfolio weights and hedge ratios. Furthermore, the effectiveness of optimal hedge ratios obtained from all considered approaches is examined.

5. Empirical outputs

The empirical findings obtained from estimating VAR-BEKK-GARCH, VAR-GARCH, VAR-AGARCH and VAR-DCC-GARCH models, which allow to investigate volatility cross effects between markets, are reported in this section. The four multivariate GARCH specifications are compared for each pair of return series. In addition, the results of diagnostic tests, constant and dynamic conditional correlations are presented.

5.1. Return, shock, and volatility interdependencies

Tables 4 and 5 (panels A to D) display detailed estimation results of four models for the stock sector index and petroleum pairs of two petroleum exporters, Canada and Saudi Arabia, and two petroleum importers, China and the United States. Starting first with the parameters of the mean equation, one can note that there are some signs of interdependencies between returns of the petroleum and stock sector indices. Specifically, outputs of the considered models indicate that past petroleum returns have statistically significant power to explain the current period returns of all sector indices in the case of Saudi Arabia and China, except the Consumer Cyclicals and Consumer Non-Cyclicals sector indices, and the Energy sector index of Canada. This finding supports the view that petroleum prices can directly or indirectly impact the cash flows of corporations (Jones & Kaul, 1996; Maghyereh et al., 2017; Degiannakis et al., 2018). The models examined reveal that the effects in the opposite direction are statistically significant for all sector indices of the developed petroleum exporter, Canada, excluding the Consumer Non-Cyclicals sector, and the petroleum importer, the United States, which suggests the potential utilisation of these sector indices' past returns as a means of forecasting the direction of petroleum returns. In the case of Saudi Arabia and China, where foreign investors have limited ownership of local stocks, the results are insignificant.

The empirical findings obtained from four multivariate GARCH models provide mixed evidence of shock or innovation and volatility interdependencies between petroleum and stock sector indices of two petroleum-exporting countries. One can observe that the cross-spillover effects are notable in the case of Canada, where the conditional volatility of petroleum is affected by shocks or innovations originating in most stock sector indices, excluding the Consumer Cyclicals sector. This outcome emphasises the importance of news emanating from these sectors of Canada for the petroleum market. Several empirical studies have identified a similar pattern in major petroleum-exporting countries (Lin et al., 2014; Belhassine & Karamti, 2021, among others). The effects in the reverse direction are generally insignificant. Bidirectional volatility spillovers are detected between the financial and industrial stock sector indices and petroleum. There is evidence of the unidirectional transmission of volatility from petroleum to the Energy stock sector index in the case of the VAR-AGARCH and VAR-DCC-GARCH models, and from the Consumer Cyclicals stock sector index to petroleum in the case of the VAR-AGARCH model. For Saudi Arabia, the estimates of all multivariate GARCH models, with the exception of VAR-BEKK-GARCH, show no signs of spillover effects. The local rules related to the ownership limits of the country's quoted shares by foreign investors that were relaxed in 2019 (for details see: Reuters, https://www.reuters.com/article/us-saudi-investment-idUSKCN1TR1SS), and factors not associated with the petroleum market, such as geopolitical events that markets in the Middle East are exposed to, could provide a plausible elucidation for the obtained finding (Wang & Liu, 2016). However, the VAR-BEKK-GARCH model points to instances of cross effects, with unilateral transmissions of shocks or innovations from the Basic Materials, Consumer Cyclicals, and Consumer Non-Cyclicals stock sector indices to petroleum, and bilateral between the Energy, Financials, and Industrials stock sector indices and petroleum. Additionally, the past volatility of all stock sector indices affects the conditional volatility of petroleum.

Turning to the empirical results concerning two petroleum-importing countries, the transmissions of shock or innovation and volatility between the petroleum and stock sector indices are limited for China. The VAR-BEKK-GARCH and VAR-AGARCH approaches show that shocks or innovations arising from the Basic Materials, Consumer Non-Cyclicals, and Financials stock sector indices impact the conditional volatility of petroleum. The VAR-DCC-GARCH model detected the effects in the opposite direction from the Consumer Cyclicals stock sector index to petroleum. The spillovers of volatility are unidirectional from petroleum to the Consumer Cyclicals and Financials stock sector indices, and from the Basic Materials and Consumer Non-Cyclicals stock sector indices to petroleum, although coefficients are weakly significant. The studies of Ashfaq et al. (2019) and Sarwar et al. (2019), which focus on the Chinese aggregate market index, suggest that the imposed foreign ownership restrictions could possibly serve as a protection against shocks associated with petroleum price movements. For the United States, the findings provide evidence of significant transmissions of shocks or innovations from all stock sector indices to petroleum, thereby implying that unanticipated short-term events in these sectors lead to the volatility upsurge in the petroleum market. Conversely, some evidence of shock or innovation spillovers from petroleum to the Basic Materials, Energy, and Industrials sector indices is documented. The past volatility of all stock sector indices has a significant effect on the conditional volatility of petroleum. The reverse volatility transmission is observed only from petroleum to the Energy stock sector index, as depicted by the coefficients of the VAR-BEKK-GARCH model. The observed findings are partially consistent with the studies conducted by Arouri et al. (2011b) and Salisu & Oloko (2015).

The estimated parameters of the own asymmetric effects of stock sector indices and petroleum are represented by D_1 and D_2, Respectively. For stock sector indices of both petroleum-exporting and importing countries, the asymmetric terms are positive and statistically significant in all cases, except the Consumer Non-Cyclicals stock sector index of Saudi Arabia and the stock sector indices of China. The obtained outcome suggests that negative shocks lead to a greater increase in conditional volatility than positive shocks of the same extent. Lin et al. (2014) and Salisu & Oloko (2015), although they consider aggregate market indices, report similar findings for petroleum-exporting and importing countries. In addition, the asymmetric terms of petroleum in all studied pairs are positive and statistically significant at conventional levels, thereby implying that unfavourable news in the petroleum market tends to cause higher conditional volatility compared to good news.

The necessary condition for the VAR-BEKK-GARCH model being covaiance stationary postulates that all the eigenvalues of$(A \otimes A) + (B \otimes B)$, where the Kronecker product of two matrices is denoted by $\otimes$, should be smaller than one in modulus, and the sufficient stability condition for the VAR-GARCH and VAR-DCC-GARCH model is $\alpha + \beta < 1$, and for thetwoVAR-AGARCH model is $\alpha + (D/2) + \beta < 1$ (Refer to the works of Engle & Kroner (1995), Engle (2002), Ling & McAleer (2003) and McAleer et al. (2009) for detailed information on necessary conditions.). These conditions are satisfied for all studied pairs (It is worth mentioning that for the VAR-BEKK-GARCH model, few of the eigenvalues in the case of the Basic Materials, Consumer Non-Cyclicals and Industrials series of Saudi Arabia barely exceed or equal to one, and the stability condition for the VAR-AGARCH model only in the case of the Consumer Cyclicals series of China is hardly greater than one). In addition, the estimated parameters θ_1 and θ_2 of the VAR-DCC-GARCH model are positive, statistically significant at conventional levels, and add up to a value that does not exceed unity. This points to the mean-reverting nature of dynamic conditional correlations.

The set of diagnostic tests was applied for standardised residuals and squared standardised residuals obtained from four multivariate GARCH models. The outputs show no evidence of serial correlations (the results available upon request). In those cases, where some autocorrelations remain in residuals, the computed values are low and remain below 0.096. Therefore, one can infer that the estimated multivariate GARCH models were defined appropriately. Based on the log likelihood, AIC and SBC values, including the diagnostic tests for residuals, the VAR-AGARCH and VAR-DCC-GARCH specifications are found to perform better among the considered models.

5.2. Constant and dynamic conditional correlations

The estimates of constant conditional correlations from the VAR-GARCH and VAR-AGARCH models between petroleum and stock sector indices of petroleum exporters and importers are represented by $\rho_{2,1}$. Both models generated similar correlations, which are positive and statistically significant at the 1% level. Overall, the stock sector indices of Canada and the United States have strong correlations with petroleum, whereas correlations between petroleum and stock sector indices of Saudi Arabia and China are weak. These findings are in line with the figures provided in Tables 2 and 3.

The time-varying conditional correlations between petroleum and stock sector indices of petroleum-exporting and importing countries obtained from the VAR-DCC-GARCH model are presented in Fig. 1. Focusing on the dynamic correlations is essential since they capture the evolving interactions between variables over time, thereby providing valuable insights for risk management in changing market conditions. It can be clearly seen that correlations, regardless of the country's position, fluctuate in both positive and negative zones throughout the period of analysis, although the predominance of positive values is documented. The magnitude and direction of dynamic conditional correlations differ considerably across sectors of exporters and importers, particularly during the major global events that led to imbalances in the petroleum market.

Table 4. The multivariate GARCH models' estimates for stock sector indices of petroleum exporters.

Panel A: Canada	Basic Materials & Brent				Consumer Cyclicals & Brent				Consumer Non-Cyclicals & Brent			
	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH
Mean Equation												
$\mu_{1,0}$	0.0001	0.0036	-0.0407	0.0035	0.0301*	0.0287*	0.0040	0.0309**	0.0405***	0.0375**	0.0219	0.0409***
$\phi_{1,1}$	0.0651***	0.0748***	0.0763***	0.0695***	0.0620***	0.0664***	0.0638***	0.0640***	0.0203	0.0233	0.0181	0.0233
$\phi_{1,2}$	-0.0008	-0.0089	-0.0072	-0.0041	-0.0075	-0.0101	-0.0079	-0.0084	-0.0028	-0.0044	-0.0035	-0.0042
$\mu_{2,0}$	0.0400	0.0467	0.0147	0.0436	0.0318	0.0368	0.0021	0.0322	0.0382	0.0280	0.0031	0.0288
$\phi_{2,1}$	0.0986***	0.1032***	0.0992***	0.1007***	0.1011***	0.0979***	0.0813***	0.1030***	0.0357	0.0428	0.0278	0.0409
$\phi_{2,2}$	-0.0203	-0.0173	-0.0150	-0.0181	-0.0089	0.00003	0.0054	-0.0074	0.0102	0.0147	0.0162	0.0132
Variance Equation												
$c_{1,1}$	0.2291***	0.0484***	0.0606***	0.0408***	0.1360***	0.0217***	0.0245***	0.0142**	0.1186***	0.0175***	0.0201***	0.0131**
$c_{2,1}$	-0.0156				0.0109				0.0306			
$c_{2,2}$	0.1154***	0.0140*	0.0130*	0.0099	0.1058***	0.0214*	0.0052	0.0118	0.0895**	0.0204**	0.0050	0.0122
$\alpha_{1,1}$	0.2528***	0.0610***	0.0200***	0.0632***	0.2344***	0.0654***	0.0114	0.0595***	0.2010***	0.0536***	0.0167	0.0538***
$\alpha_{1,2}$	0.0039	0.0024	0.0010	0.0018	0.0277	0.0021	0.0012	0.0022	0.0288	-0.00002	-0.0002	0.0001
$\alpha_{2,1}$	-0.0115	0.0083*	0.0012	0.0086*	0.0147	0.0291	-0.0227*	0.0236	0.0191*	0.0407*	-0.0035	0.0392
$\alpha_{2,2}$	0.1887***	0.0359***	0.0155***	0.0363***	0.1911***	0.0381***	0.0160***	0.0388***	0.1939***	0.0363***	0.0154***	0.0368***
$\beta_{1,1}$	0.9611***	0.9276***	0.9219***	0.9279***	0.9624***	0.9084***	0.9113***	0.9242***	0.9701***	0.9141***	0.9197***	0.9214***
$\beta_{1,2}$	-0.00001	-0.0014	0.0044	-0.0002	-0.0068	-0.0001	0.0019	-0.0006	-0.0124	0.0031	0.0030	0.0028
$\beta_{2,1}$	0.0062	-0.0072	-0.0004	-0.0067	-0.0008	-0.0324	0.0364*	-0.0154	-0.0016	-0.0448	0.0230	-0.0319
$\beta_{2,2}$	0.9804***	0.9596***	0.9598***	0.9599***	0.9807***	0.9576***	0.9589***	0.9567***	0.9813***	0.9597***	0.9595***	0.9593***
D_1			0.0743***				0.0846***				0.0575***	
D_2			0.0416***				0.0404***				0.0390***	
$\rho_{2,1}$		0.3278***	0.3267***			0.3147***	0.3101***			0.2785***	0.2726***	
θ_1				0.0143**				0.0198***				0.0230***
θ_2				0.9806***				0.9764***				0.9724***

Log L	-14079.265	-14078.053	-14044.649	-14058.887	-11864.552	-11893.806	-11860.706	-11852.316	-11573.022	-11614.220	-11590.120	-11564.675
AIC	8.172	8.171	8.153	8.161	6.888	6.905	6.887	6.881	6.719	6.743	6.730	6.715
SBC	8.202	8.201	8.187	8.193	6.918	6.935	6.921	6.913	6.749	6.773	6.764	6.747

Panel B: Canada	Energy & Brent				Financials & Brent				Industrials & Brent			
	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH
Mean Equation												
$\mu_{1,0}$	0.0151	0.0236	-0.0222	0.0188	0.0316***	0.0296**	0.0092	0.0328***	0.0381**	0.0387**	0.0152	0.0405**
$\phi_{1,1}$	0.1193***	0.1331***	0.1369***	0.1311***	0.0924***	0.1001***	0.0952***	0.0968***	0.0661***	0.0760***	0.0702***	0.0710***
$\phi_{1,2}$	-0.0267	-0.0373**	-0.0380**	-0.0301*	-0.0052	-0.0098	-0.0081	-0.0076	-0.0020	-0.0084	-0.0074	-0.0068
$\mu_{2,0}$	0.0445	0.0520**	0.0197	0.0481*	0.0375	0.0341	0.0051	0.0354	0.0319	0.0338	0.0032	0.0376
$\phi_{2,1}$	0.2174***	0.2263***	0.2267***	0.2246***	0.1128***	0.1144***	0.1023***	0.1191***	0.1035***	0.1063***	0.0956***	0.1065***
$\phi_{2,2}$	-0.0873***	-0.0923***	-0.0913***	-0.0862***	-0.0101	-0.0050	-0.0027	-0.0092	-0.0075	-0.0045	-0.0008	-0.0057
Variance Equation												
$c_{1,1}$	0.1843***	0.0291**	0.0284**	0.0184*	-0.0977***	0.0075***	0.0071***	0.0044*	0.1680***	0.0247***	0.0238***	0.0189***
$c_{2,1}$	-0.0247				-0.0163				0.0005			
$c_{2,2}$	0.1400*	0.0189***	0.0193***	0.0107*	0.0967***	0.0190***	0.0161***	0.0141**	0.0805**	0.0269***	0.0149**	0.0197***
$\alpha_{1,1}$	0.2972***	0.0804***	-0.0019	0.0791***	0.2709***	0.0894***	0.0280***	0.0875***	0.2823***	0.0894***	0.0199*	0.0872***
$\alpha_{1,2}$	-0.0019	0.0022	-0.0001	0.0017	0.0552	-0.0007	-0.0002	-0.0007	0.0675*	-0.0026	-0.0038	-0.0026
$\alpha_{2,1}$	-0.0204	0.0169**	0.0128*	0.0163**	0.0084	0.0447***	0.0139	0.0464***	-0.0101	0.0516**	0.0117	0.0490***
$\alpha_{2,2}$	0.1818***	0.0304***	0.0192***	0.0308***	0.1834***	0.0349***	0.0174***	0.0365***	0.1769***	0.0338***	0.0166***	0.0351***
$\beta_{1,1}$	0.9537***	0.8835***	0.8831***	0.8906***	0.9574***	0.8894***	0.9094***	0.8979***	0.9468***	0.8625***	0.8725***	0.8746***
$\beta_{1,2}$	0.0154	0.0219	0.0387**	0.0219*	-0.0136	0.0047*	0.0041*	0.0044	-0.0188	0.0118*	0.0144***	0.0110***
$\beta_{2,1}$	0.0021	-0.0129	-0.0079	-0.0092	-0.0001	-0.0492***	-0.0116	-0.0459**	0.0057	-0.0723**	-0.0115	-0.0589**
$\beta_{2,2}$	0.9735***	0.9605***	0.9582***	0.9609***	0.9825***	0.9616***	0.9603***	0.9609***	0.9844***	0.9661***	0.9626***	0.9642***
D_1			0.1220***				0.0776***				0.1028***	
D_2			0.0251***				0.0352***				0.0340***	
$\rho_{2,1}$		0.4972***	0.4969***			0.3459***	0.3406***			0.3399***	0.3339***	

θ_1				0.0143***				0.0277***				0.0229***
θ_2				0.9841***				0.9677***				0.9725***
Log L	-13215.578	-13208.297	-13169.957	-13173.381	-11514.308	-11560.269	-11533.894	-11492.110	-11955.699	-11982.813	-11951.417	-11933.589
AIC	7.671	7.667	7.646	7.647	6.685	6.711	6.697	6.673	6.941	6.956	6.939	6.928
SBC	7.701	7.697	7.680	7.679	6.715	6.742	6.731	6.705	6.971	6.987	6.973	6.961

Panel C: Saudi Arabia	Basic Materials & Brent				Consumer Cyclicals & Brent				Consumer Non-Cyclicals & Brent			
	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH
Mean Equation												
$\mu_{1,0}$	0.0627***	0.0553***	0.0374**	0.0591***	0.0413**	0.0433**	0.0312*	0.0476**	0.0609*	0.0527**	0.0452*	0.0576**
$\phi_{1,1}$	0.0806***	0.0940***	0.0994***	0.0860***	0.1025***	0.1136***	0.1179***	0.1100***	0.1032***	0.1063***	0.1065***	0.1007***
$\phi_{1,2}$	0.0462***	0.0494***	0.0445***	0.0499***	0.0157	0.0290***	0.0269***	0.0285***	0.0232**	0.0328**	0.0314***	0.0326***
$\mu_{2,0}$	0.0645**	0.0401	0.0111	0.0506	0.0394	0.0449	0.0159	0.0582	0.0698**	0.0475	0.0193	0.0586
$\phi_{2,1}$	0.0050	0.0104	0.0031	0.0140	0.0150	0.0134	0.0077	0.0162	-0.0104	-0.0062	-0.0130	-0.0030
$\phi_{2,2}$	0.0215	0.0227	0.0210	0.0203	0.0112	0.0227	0.0219	0.0161	0.0185	0.0234	0.0224	0.0206
Variance Equation												
$c_{1,1}$	0.2161***	0.0408*	0.0392	0.0388	0.2516***	0.0406*	0.0405**	0.0395**	0.2940***	0.0777*	0.0752	0.0758**
$c_{2,1}$	-0.0847***				-0.0644				-0.0849**			
$c_{2,2}$	0.0919***	0.0122**	0.0077	0.0105	0.0986***	0.0124**	0.0078	0.0111*	0.0886***	0.0127**	0.0074	0.0115*
$\alpha_{1,1}$	0.4042***	0.1682***	0.0928***	0.1697***	0.4028***	0.1756***	0.1434***	0.1793***	0.4467***	0.2096***	0.1791***	0.2116***
$\alpha_{1,2}$	-0.0282*	0.0057	0.0035	0.0064	-0.0301**	0.0123	0.0095	0.0132	-0.0322***	0.0194	0.0158	0.0204
$\alpha_{2,1}$	0.0358	0.0067	0.0055	0.0070	-0.0109	0.0078	0.0063	0.0081	0.0667	0.0059	0.0041	0.0058
$\alpha_{2,2}$	0.1568***	0.0333***	0.0086	0.0342***	0.1665***	0.0350***	0.0098**	0.0358***	0.1571***	0.0361***	0.0094**	0.0372***
$\beta_{1,1}$	0.9115***	0.8258***	0.8295***	0.8230***	0.9127***	0.8065***	0.8068***	0.8017***	0.8937***	0.7875***	0.7780***	0.7846***
$\beta_{1,2}$	0.0176***	0.0002	0.0034	0.0012	0.0152***	0.0033	0.0060	0.0040	0.0162***	-0.0059	-0.0016	-0.0052
$\beta_{2,1}$	0.0023	-0.0040	-0.0034	-0.0045	0.0073	-0.0065	-0.0054	-0.0070	0.0010	-0.0046	-0.0030	-0.0046
$\beta_{2,2}$	0.9835***	0.9625***	0.9681***	0.9625***	0.9832***	0.9617***	0.9672***	0.9618***	0.9840***	0.9606***	0.9665***	0.9603***
D_1			0.1254***				0.0581**				0.0847	

D_2			0.0411***				0.0423***				0.0439***	
$\rho_{2,1}$		0.1583***	0.1513***			0.1047***	0.0961***			0.1023***	0.0949***	
θ_1				0.0144**				0.0134*				0.0135**
θ_2				0.9820***				0.9840***				0.9839***
Log L	-12897.486	-12901.378	-12862.424	-12881.830	-13154.375	-13143.427	-13119.797	-13122.804	-13647.264	-13643.611	-13617.418	-13623.929
AIC	7.550	7.552	7.531	7.542	7.700	7.694	7.681	7.682	7.988	7.986	7.972	7.975
SBC	7.581	7.583	7.565	7.574	7.731	7.724	7.715	7.715	8.019	8.017	8.006	8.008

Panel D: Saudi Arabia	Energy & Brent				Financials & Brent				Industrials & Brent			
	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH
Mean Equation												
$\mu_{1,0}$	0.0626***	0.0465*	0.0320	0.0482**	0.0383	0.0197	-0.0046	0.0254	0.0475**	0.0415*	0.0251	0.0446**
$\phi_{1,1}$	0.0798***	0.0864***	0.0895***	0.0827***	0.1515***	0.1599***	0.1817***	0.1548***	0.1189***	0.1254***	0.1248***	0.1204***
$\phi_{1,2}$	0.0306***	0.0311**	0.0286**	0.0314***	0.0169	0.0230**	0.0193*	0.0226	0.031***	0.0395***	0.0357***	0.0393***
$\mu_{2,0}$	0.0578*	0.0370	0.0091	0.0457	0.0571	0.0359	0.0077	0.0416	0.0692**	0.0439*	0.0137	0.0509*
$\phi_{2,1}$	-0.0003	-0.0001	-0.0071	0.0009	-0.0173	-0.0058	-0.0115	-0.0057	-0.0064	0.0008	-0.0055	0.0035
$\phi_{2,2}$	0.0182	0.0226	0.0213	0.0204	0.0191	0.0213	0.0184	0.0193	0.0205	0.0247	0.0224	0.0222
Variance Equation												
$c_{1,1}$	0.2788***	0.0651**	0.0650**	0.0640***	0.3679***	0.1093	0.1405**	0.1112	0.2997***	0.0709	0.0712	0.0697*
$c_{2,1}$	-0.0598**				-0.1005**				-0.0863***			
$c_{2,2}$	0.1031***	0.0122*	0.0077	0.0110**	0.0925***	0.0108	0.0045	0.0107	0.0956***	0.0121*	0.0073	0.0111*
$\alpha_{1,1}$	0.3883***	0.1351***	0.0964***	0.1340***	0.3889***	0.1431***	0.0312	0.1463***	0.4079***	0.1785***	0.1313***	0.1787***
$\alpha_{1,2}$	-0.0336***	0.0074	0.0042	0.0080	-0.0331**	0.0066	-0.0005	0.0072	-0.0366***	0.0136	0.0078	0.0141
$\alpha_{2,1}$	0.0487*	0.0057	0.0045	0.0052	0.0503*	0.0122	0.0067	0.0128	0.0694*	0.0057	0.0036	0.0057
$\alpha_{2,2}$	0.1611***	0.0348***	0.0089*	0.0356***	0.1473***	0.0296***	0.0063	0.0297***	0.1520***	0.0347***	0.0093*	0.0352***
$\beta_{1,1}$	0.9141***	0.8539***	0.8547***	0.8537***	0.8972***	0.8084***	0.7462***	0.8031***	0.9026***	0.8008***	0.7951***	0.7993***
$\beta_{1,2}$	0.0179***	-0.0040	-0.0013	-0.0036	0.0263**	0.0056	0.0324	0.0067	0.0203***	0.0038	0.0102	0.0046

$\beta_{2,1}$	-0.0010	-0.0039	-0.0030	-0.0036	0.0010	-0.0083	-0.0013	-0.0093	-0.0002	-0.0043	-0.0023	-0.0044
$\beta_{2,2}$	0.9829***	0.9610***	0.9669***	0.9610***	0.9839***	0.9654***	0.9673***	0.9659***	0.9836***	0.9617***	0.9669***	0.9618***
D_1			0.0714*				0.2399**				0.0974**	
D_2			0.0432***				0.0442***				0.0428***	
$\rho_{2,1}$		0.1348***	0.1277***			0.1051***	0.0997***			0.1254***	0.1177***	
θ_1				0.0087*				0.0103**				0.0090*
θ_2				0.9880***				0.9856***				0.9872***
Log L	-13563.326	-13564.287	-13535.921	-13550.571	-13288.558	-13277.631	-13215.456	-13267.067	-13535.352	-13536.110	-13506.729	-13521.857
AIC	7.939	7.940	7.925	7.933	7.779	7.772	7.737	7.767	7.923	7.923	7.907	7.916
SBC	7.970	7.970	7.959	7.965	7.809	7.803	7.771	7.799	7.954	7.954	7.942	7.948

Notes: The models are estimated by the QMLE method using the BFGS algorithm and robust standard errors. The stock sector indices are ordered as (1) and Brent crude petroleum as (2). In the mean equation, μ denotes constant terms and φ represents autoregressive terms with one lag. For instance, the coefficient $\phi_{1,2}$ captures the impact of one period lagged petroleum returns on current period returns of stock sector indices. In the variance equation, c refers to constant terms, α and β are ARCH and GARCH terms, respectively. For example, the coefficients $\alpha_{1,2}$ and $\beta_{1,2}$ in the VAR-GARCH, VAR-AGARCH and VAR-DCC-GARCH models measure the transmission of innovations or shocks and volatilities from petroleum to stock sector indices. The interpretation of parameters $\alpha_{1,2}$ and $\beta_{1,2}$ in the VAR-BEKK-GARCH model is opposite, that is, the direction of spillovers is from stock sector indices to petroleum. In addition, D represents asymmetric effects, ϱ refers to constant conditional correlations, θ_1 and θ_2 are the parameters of dynamic conditional correlations.

Table 5. The multivariate GARCH models' estimates for stock sector indices of petroleum importers.

Panel A: China	Basic Materials & Brent				Consumer Cyclicals & Brent				Consumer Non-Cyclicals & Brent			
	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH
Mean Equation												
$\mu_{1,0}$	-0.0006	0.0125	0.0092	0.0088	0.0055	0.0108	0.0160	0.0090	0.0258	0.0369	0.0426*	0.0352**
$\phi_{1,1}$	0.0429***	0.0482**	0.0473**	0.0447***	0.0403**	0.0439**	0.0431**	0.0427**	0.0506***	0.0521**	0.0505**	0.0508***
$\phi_{1,2}$	0.0757***	0.0694***	0.0695***	0.0716***	0.0229*	0.0202	0.0216	0.0212	0.0116	0.0093	0.0103	0.0102
$\mu_{2,0}$	0.0523*	0.0366	0.0044	0.0443	0.0378	0.0360	0.0057	0.0395	0.0400	0.0380	0.0055	0.0405
$\phi_{2,1}$	0.0032	0.0086	0.0145	0.0089	0.0108	0.0111	0.0147	0.0120	-0.0003	0.0033	0.0046	0.0027
$\phi_{2,2}$	0.0147	0.0195	0.0236	0.0172	0.0142	0.0205	0.0241	0.0195	0.0174	0.0220	0.0258	0.0206
Variance Equation												
$c_{1,1}$	-0.1863***	0.0340***	0.0308***	0.0331***	0.1287***	0.0151***	0.0137**	0.0148***	0.1549***	0.0193**	0.0166**	0.0190**
$c_{2,1}$	0.0542*				0.0090				-0.0426			
$c_{2,2}$	-0.0918***	0.0058	-0.0030	0.0055	0.0920**	0.0073	0.0008	0.0072	-0.0922***	0.0062	-0.0015	0.0061
$\alpha_{1,1}$	0.2358***	0.0609***	0.0602***	0.0605***	0.2238***	0.0483***	0.0555***	0.0479***	0.2424***	0.0624***	0.0667***	0.0622***
$\alpha_{1,2}$	-0.0345**	-0.0009	-0.0031	-0.0010	0.0211	0.0054	0.0043	0.0054**	-0.0396*	0.0037	0.0026	0.0037
$\alpha_{2,1}$	0.0107	0.0012	-0.0043	0.0010	-0.0039	0.0009	-0.0020	0.0007	0.0042	-0.0014	-0.0065*	-0.0016
$\alpha_{2,2}$	0.1783***	0.0353***	0.0039	0.0352***	0.1703***	0.0360***	0.0075*	0.0360***	0.1752***	0.0356***	0.0048	0.0356***
$\beta_{1,1}$	0.9682***	0.9337***	0.9346***	0.9342***	0.9731***	0.9509***	0.9520***	0.9514***	0.9674***	0.9339***	0.9377***	0.9342***
$\beta_{1,2}$	0.0108**	-0.0002	0.0030	-0.0001	-0.0060	-0.0066*	-0.0059	-0.0066***	0.0117*	-0.0036	-0.0027	-0.0035
$\beta_{2,1}$	-0.0009	0.0023	0.0090*	0.0025	0.0020	0.0016	0.0053	0.0017	-0.0002	0.0048	0.0111**	0.0050
$\beta_{2,2}$	0.9818***	0.9601***	0.9667***	0.9603***	0.9847***	0.9607***	0.9669***	0.9608***	0.9826***	0.9606***	0.9680***	0.9607***
D_1			-0.0013				-0.0138				-0.0134	
D_2			0.0519***				0.0465***				0.0488***	
$\rho_{2,1}$		0.1035***	0.1038***			0.0814***	0.0819***			0.0691***	0.0695***	
θ_1				0.0070**				0.0059*				0.0081***
θ_2				0.9906***				0.9892***				0.9820***
Log L	-13826.358	-13814.063	-13783.506	-13805.153	-13519.635	-13497.416	-13469.538	-13493.745	-13383.453	-13362.923	-13332.896	-13359.646

AIC	8.287	8.280	8.262	8.275	8.103	8.090	8.075	8.088	8.022	8.010	7.993	8.008
SBC	8.318	8.311	8.297	8.308	8.134	8.121	8.109	8.121	8.053	8.041	8.028	8.041

Panel B: China	Energy & Brent				Financials & Brent				Industrials & Brent			
	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH
Mean Equation												
$\mu_{1,0}$	-0.0191	-0.0087	-0.0100	-0.0131	0.0092	0.0086	0.0093	0.0074	-0.0101	-0.0041	-0.0034	-0.0073
$\phi_{1,1}$	0.0246	0.0301	0.0308	0.0273*	0.0178	0.0206	0.0225	0.0185	0.0369**	0.0411**	0.0412**	0.0392**
$\phi_{1,2}$	0.0935***	0.0865***	0.0879***	0.0884***	0.0394**	0.0369**	0.0368**	0.0383***	0.0345***	0.0312**	0.0321***	0.0331**
$\mu_{2,0}$	0.0462	0.0353	0.0038	0.0415	0.0434	0.0300	0.0049	0.0383	0.0499	0.0340	0.0050	0.0423
$\phi_{2,1}$	0.0082	0.0119	0.0181	0.0123	0.0069	0.0031	0.0065	0.0044	0.0043	0.0078	0.0136	0.0072
$\phi_{2,2}$	0.0127	0.0194	0.0223	0.0156	0.0164	0.0203	0.0245	0.0179	0.0134	0.0204	0.0248	0.0173
Variance Equation												
$c_{1,1}$	0.1776***	0.0312***	0.0290***	0.0299***	0.1173***	0.0184***	0.0174***	0.0180***	0.1414***	0.0205**	0.0193**	0.0198***
$c_{2,1}$	-0.0491				-0.0017				-0.0362			
$c_{2,2}$	0.0982***	0.0081	-0.0020	0.0073	0.0842**	0.0063	-0.0001	0.0059	-0.1021***	0.0067	0.0006	0.0061
$\alpha_{1,1}$	0.2331***	0.0559***	0.0586***	0.0555***	0.2045***	0.0389***	0.0417***	0.0386***	0.2082***	0.0457***	0.0481***	0.0452***
$\alpha_{1,2}$	-0.0129	0.0004	-0.0026	0.0002	0.0362*	0.0023	0.0006	0.0024	-0.0171	0.0013	-0.0007	0.0012
$\alpha_{2,1}$	-0.0137	0.0049	0.0003	0.0045	-0.0029	0.0075	0.0003	0.0071	0.0018	0.0044	-0.0011	0.0039
$\alpha_{2,2}$	0.1791***	0.0351***	0.0043	0.0352***	0.1714***	0.0329***	0.0078*	0.0334***	0.1813***	0.0345***	0.0065	0.0344***
$\beta_{1,1}$	0.9698***	0.9406***	0.9410***	0.9412***	0.9776***	0.9609***	0.9596***	0.9613***	0.9756***	0.9508***	0.9505***	0.9514***
$\beta_{1,2}$	0.0052	-0.0022	0.0014	-0.0018	-0.0080	-0.0057**	-0.0035	-0.0056***	0.0044	-0.0024	0.0000	-0.0022
$\beta_{2,1}$	0.0030	-0.0020	0.0040	-0.0016	0.0016	-0.0029	0.0041	-0.0025	0.0007	0.0001	0.0061	0.0006
$\beta_{2,2}$	0.9823***	0.9603***	0.9665***	0.9604***	0.9843***	0.9615***	0.9661***	0.9613***	0.9822***	0.9605***	0.9658***	0.9608***
D_1			-0.0062				-0.0032				-0.0043	
D_2			0.0515***				0.0453***				0.0482***	
$\rho_{2,1}$		0.1207***	0.1209***			0.0732***	0.0722***			0.0810***	0.0818***	
θ_1				0.0064***				0.0131*				0.0137**
θ_2				0.9922***				0.9735***				0.9758***

Log L	-13978.511	-13968.444	-13938.709	-13958.866	-13659.072	-13639.263	-13617.271	-13633.307	-13485.474	-13473.548	-13447.090	-13465.131
AIC	8.378	8.372	8.355	8.367	8.187	8.175	8.163	8.172	8.083	8.076	8.061	8.071
SBC	8.409	8.403	8.390	8.400	8.218	8.206	8.198	8.205	8.114	8.107	8.096	8.104

Panel C: United States	Basic Materials & Brent				Consumer Cyclicals & Brent				Consumer Non-Cyclicals & Brent			
	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH
Mean Equation												
$\mu_{1,0}$	0.0435**	0.0422**	0.0009	0.0441***	0.0366**	0.0317**	0.0055	0.0342**	0.0460***	0.0427***	0.0204*	0.0442***
$\phi_{1,1}$	-0.0190	-0.0146	-0.0100	-0.0171	-0.0021	0.0143	0.0181	0.0046	-0.0584***	-0.0440***	-0.0420**	-0.0513***
$\phi_{1,2}$	0.0110	0.0087	0.0104	0.0105	0.0022	0.0008	0.0008	0.0016	0.0011	0.0002	-0.0003	0.0006
$\mu_{2,0}$	0.0343	0.0341	-0.0011	0.0371	0.0178	0.0288	0.0002	0.0225	0.0277	0.0279	-0.0010	0.0238
$\phi_{2,1}$	0.1669***	0.1685***	0.1645***	0.1668***	0.1187***	0.1162***	0.1065***	0.1188***	0.1364***	0.1305***	0.1242***	0.1313***
$\phi_{2,2}$	-0.0155	-0.0086	-0.0066	-0.0135	-0.0019	0.0091	0.0108	0.0007	0.0109	0.0138	0.0140	0.0114
Variance Equation												
$c_{1,1}$	0.1740***	0.0297***	0.0317***	0.0272***	0.1472***	0.0208***	0.0219***	0.0202***	0.1430***	0.0199***	0.0210***	0.0193***
$c_{2,1}$	0.0040				0.0231				0.0597**			
$c_{2,2}$	0.0947***	0.0180***	0.0129**	0.0163***	0.0928***	0.0159**	0.0093	0.0159***	0.0725**	0.0223**	0.0090	0.0217**
$\alpha_{1,1}$	0.2907***	0.0908***	0.0002	0.0916***	0.2860***	0.0866***	0.0219**	0.0861***	0.3163***	0.1119***	0.0081	0.1116***
$\alpha_{1,2}$	0.0322	0.0043*	0.0018	0.0044*	0.0460*	0.0014	0.0006	0.0016	0.0885**	0.0011	0.0004	0.0013
$\alpha_{2,1}$	0.0109	0.0279***	0.0153	0.0292***	0.0038	0.0315**	0.0089	0.0318***	0.0095	0.0901***	0.0325	0.0889***
$\alpha_{2,2}$	0.1853***	0.0365***	0.0168***	0.0374***	0.1798***	0.0361***	0.0163***	0.0367***	0.1852***	0.0376***	0.0152***	0.0381***
$\beta_{1,1}$	0.9474***	0.8911***	0.9051***	0.8927***	0.9502***	0.8981***	0.9068***	0.8993***	0.9322***	0.8547***	0.8701***	0.8565***
$\beta_{1,2}$	-0.0099	-0.0030	0.0012	-0.0028	-0.0153**	-0.0011	-0.0002	-0.0011	-0.0405**	-0.0003	0.0006	-0.0004
$\beta_{2,1}$	0.0010	-0.0292**	-0.0147	-0.0291***	0.0004	-0.0325*	-0.0064	-0.0324**	-0.0011	-0.1056**	-0.0309	-0.1013**
$\beta_{2,2}$	0.9824***	0.9598***	0.9617***	0.9592***	0.9831***	0.9605***	0.9613***	0.9599***	0.9823***	0.9601***	0.9636***	0.9594***
D_1			0.1314***				0.1046***				0.1595***	
D_2			0.0368***				0.0394***				0.0391***	
$\rho_{2,1}$		0.2334***	0.2276***			0.1287***	0.1227***			0.0915***	0.0810***	
θ_1				0.0242**				0.0223***				0.0152***
θ_2				0.9685***				0.9729***				0.9824***

Log L	-12610.240	-12629.010	-12580.945	-12590.874	-12238.742	-12281.217	-12238.000	-12220.712	-10960.575	-10987.586	-10935.983	-10940.378
AIC	7.303	7.314	7.287	7.293	7.088	7.113	7.089	7.078	6.349	6.365	6.336	6.338
SBC	7.333	7.344	7.321	7.325	7.119	7.143	7.123	7.110	6.379	6.395	6.370	6.370

Panel D: United States	Energy & Brent				Financials & Brent				Industrials & Brent			
	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH	VAR-BEKK-GARCH	VAR-GARCH	VAR-AGARCH	VAR-DCC-GARCH
Mean Equation												
$\mu_{1,0}$	0.0313	0.0437*	0.0060	0.0419**	0.0537***	0.0520***	0.0202	0.0536***	0.0619***	0.0575***	0.0217	0.0608***
$\phi_{1,1}$	-0.0275	-0.0288*	-0.0246	-0.0268	-0.0736***	-0.0630***	-0.0525***	-0.0681***	-0.0331**	-0.0253*	-0.0148	-0.0302
$\phi_{1,2}$	0.0174	0.0122	0.0141	0.0158	0.0026	0.0021	0.0018	0.0016	0.0068	0.0057	0.0072	0.0064
$\mu_{2,0}$	0.0199	0.0357	0.0055	0.0368	0.0307	0.0353	0.0048	0.0291	0.0291	0.0316	-0.0043	0.0322
$\phi_{2,1}$	0.2491***	0.2532***	0.2529***	0.2501***	0.1037***	0.0987***	0.0912***	0.1072***	0.1273***	0.1311***	0.1245***	0.1292***
$\phi_{2,2}$	-0.0754***	-0.0790***	-0.0771***	-0.0748***	-0.0049	0.0106	0.0116	-0.0017	-0.0039	0.0036	0.0064	-0.0031
Variance Equation												
$c_{1,1}$	0.1785***	0.0353***	0.0430***	0.0300**	0.1455***	0.0216***	0.0211***	0.0209***	0.1564***	0.0266***	0.0248***	0.0251***
$c_{2,1}$	0.0115				0.0058				0.0219			
$c_{2,2}$	0.0488	0.0161***	0.0159***	0.0128**	0.0952***	0.0137***	0.0099**	0.0138***	0.0926***	0.0185***	0.0088	0.0175**
$\alpha_{1,1}$	0.2971***	0.0811***	0.0206*	0.0792***	0.3471***	0.1275***	0.0374***	0.1278***	0.2882***	0.0913***	-0.0168*	0.0924***
$\alpha_{1,2}$	0.0859**	0.0054	0.0025	0.0045	0.0326	0.0007	0.0005	0.0008	0.0454	0.0037*	0.0012	0.0039
$\alpha_{2,1}$	-0.0701***	0.0203**	0.0234***	0.0205**	0.0088	0.0205**	0.0096	0.0222***	0.0129	0.0310**	-0.0007	0.0322*
$\alpha_{2,2}$	0.1353***	0.0304***	0.0129**	0.0306***	0.1825***	0.0364***	0.0157***	0.0368***	0.1896***	0.0386***	0.0168***	0.0398***
$\beta_{1,1}$	0.9493***	0.8980***	0.8794***	0.9008***	0.9331***	0.8647***	0.8827***	0.8652***	0.9468***	0.8873***	0.9159***	0.8882***
$\beta_{1,2}$	-0.0256*	0.0041	0.0235	0.0057	-0.0109	-0.0006	-0.0002	-0.0006	-0.0157*	-0.0032	-0.0006	-0.0032
$\beta_{2,1}$	0.0182**	-0.0162	-0.0242**	-0.0164	0.0004	-0.0198**	-0.0083	-0.0212**	-0.0006	-0.0345*	0.0030	-0.0340*
$\beta_{2,2}$	0.9928***	0.9616***	0.9686***	0.9627***	0.9827***	0.9603***	0.9623***	0.9598***	0.9816***	0.9583***	0.9605***	0.9572***
D_1			0.1057***				0.1312***				0.1499***	
D_2			0.0301***				0.0388***				0.0409***	

$\rho_{2,1}$		0.4304***	0.4262***			0.1564***	0.1476***			0.1764***	0.1719***	
θ_1				0.0093***				0.0267***				0.0295***
θ_2				0.9890***				0.9686***				0.9644***
Log L	-13326.482	-13304.888	-13279.088	-13281.628	-12396.468	-12442.374	-12397.145	-12375.882	-12050.842	-12101.359	-12033.057	-12038.357
AIC	7.717	7.705	7.691	7.692	7.180	7.206	7.181	7.168	6.980	7.009	6.971	6.973
SBC	7.748	7.735	7.725	7.724	7.210	7.236	7.215	7.200	7.010	7.039	7.004	7.005

Notes: The models are estimated by the QMLE method using the BFGS algorithm and robust standard errors. The stock sector indices are ordered as (1) and Brent crude petroleum as (2). In the mean equation, μ denotes constant terms and φ represents autoregressive terms with one lag. For instance, the coefficient $\phi_{1,2}$ captures the impact of one period lagged petroleum returns on current period returns of stock sector indices. In the variance equation, c refers to constant terms, α and β are ARCH and GARCH terms, respectively. For example, the coefficients $\alpha_{1,2}$ and $\beta_{1,2}$ in the VAR-GARCH, VAR-AGARCH and VAR-DCC-GARCH models measure the transmission of innovations or shocks and volatilities from petroleum to stock sector indices. The interpretation of parameters $\alpha_{1,2}$ and $\beta_{1,2}$ in the VAR-BEKK-GARCH model is opposite, that is, the direction of spillovers is from stock sector indices to petroleum. In addition, D represents asymmetric effects, ϱ refers to constant conditional correlations, θ_1 and θ_2 are the parameters of dynamic conditional correlations.

Starting with the petroleum exporting countries, the Energy and Consumer Non-Cyclicals stock sector indices of Canada exhibited high and low correlations with petroleum, respectively. Furthermore, conditional correlations do not enter negative regions for the Basic Materials and Energy stock sector indices. In the case of Saudi Arabia, correlations for all stock sector indices mostly move in tandem and have similar levels. The first period of interest from 2005 to 2009 is associated with rising petroleum demand, especially by emerging economies, despite the stagnation in production, and the global financial crisis, which resulted in unprecedented petroleum prices. The conditional correlations for stock sector indices of Canada remain positive and generally stable at the start of this period. On the other hand, stock sector indices of Saudi Arabia show low conditional correlations that turn negative from mid-2005 until mid-2006. This finding could be elucidated by a surge in volatilities of stock sector indices due to the rise and collapse of the country's stock market (see Fig. A.2 in Appendix). During the global financial crisis, conditional correlations for stock sector indices of both countries experienced intense fluctuations (Fig. A.1 and Fig. A.2 in the Appendix depict spikes in volatilities of petroleum prices and stock sector indices during the period 2008-2009). The continued growth of petroleum prices amid the negative effects of the recession on the stock markets provides a plausible explanation for the downward trend in correlations by the middle of 2008. It is worth noting that despite the drop, values remain positive for the Basic Materials and Energy stock sector indices of Canada, and the Basic Materials, Energy, and Industrials stock sector indices of Saudi Arabia. The dynamic conditional correlations between the petroleum and stock sector indices increased dramatically by the end of 2008. This upward and positive trend is because the global financial crisis caused uncertainty in the stock markets and led to a substantial decline in petroleum prices (Filis et al., 2011; Boldanov et al., 2016).

The second period of interest, from 2010 to 2013, is associated with the Arab Spring and subsequent unrest in the petroleum-rich MENA region. The conditional correlations were positive and high at the beginning of this period. A noticeable decline in values is detected in the first half of 2011. During this period, the rising pattern in petroleum prices was not followed by analogous movements in stock sector indices. The conditional correlations dropped, but remained positive for the stock sector indices of Canada. Boldanov et al. (2016) also report declines, with values reaching a negative level, which could be attributed to differences in methodological frameworks employed. In the case of Saudi Arabia, conditional correlations between petroleum and stock sector indices took a negative form, which is possibly related to spikes in volatilities of stock sector indices caused by the growing instabilities in the region (see Fig. A.2 in the Appendix). This observation corroborates the finding of Maghyereh et al. (2017), where the aggregate stock market index is utilised. The increase in conditional correlations by the end of 2011 could be attributed to pessimistic economic events, such as the Eurozone debt crisis and the downgrade of the United States' credit rating, which tumbled stock markets across the world. The fluctuations of conditional correlations in 2012 are explained by the moderate upward and downward trends in petroleum prices linked to concerns around potential supply disruptions and changing expectations on global economic activity. The conditional correlations for stock sector indices of both countries gradually declined by the end of 2013 due to the relative stability in petroleum prices.

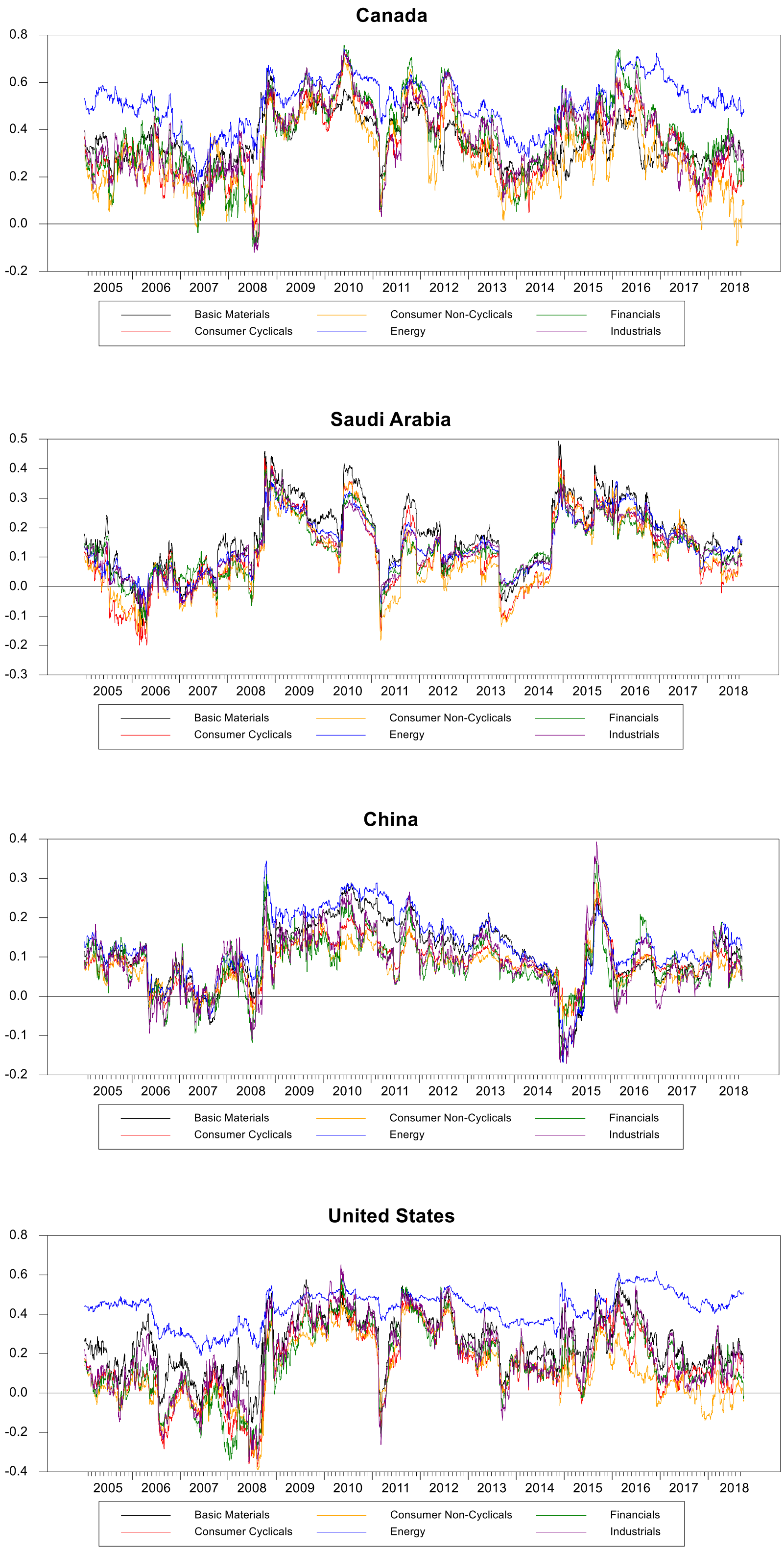
One can observe interesting patterns during the final period from 2014 to 2018 that are associated with the geopolitical crisis. The collapse of petroleum prices escalated volatility in the first half of this period. The decline in the price level resulted from several reasons. First, crude petroleum production, particularly non-OPEC, steadily increased. The OPEC countries did not respond to this growth but maintained their production levels. Second, the weak global economic activity led to a reduction in petroleum demand (Baumeister & Kilian, 2016). Furthermore, geopolitical tensions fuelled petroleum price volatility. Fig. A.1 and Fig. A.2 in Appendix illustrate that volatilities of stock sector indices, albeit with different magnitudes, also surged during this sub-period. Starting from 2014, time-varying conditional correlations displayed an increasing trend for stock sector indices of Canada and reached a peak in the first half of 2016. For Saudi Arabia, dynamic conditional correlations of stock sector indices with petroleum climbed sharply in the second half of 2014 and remained high until the end of 2016. Yousaf et al. (2022) also observe a similar trend during this period. The conditional correlations for stock sector indices of both petroleum-exporting countries stabilised during the period 2017-2018.

Turning to the petroleum-importing countries, conditional correlations between petroleum stock sector indices of China show similar movements over the sample period. As in the case of Canada, the largest and lowest conditional correlations are detected for the Energy and Consumer Non-Cyclicals stock sector indices of the United States. In addition, only correlations for the Energy stock sector index do not enter negative territories. At the beginning of the first period 2005-2009, time-varying conditional correlations for most stock sector indices of China and the United States are relatively weak and swung in both positive and negative ranges. The conditional correlations of all stock sector indices with petroleum resemble those of their counterparts from the petroleum exporting countries during the global financial crisis. Specifically, the values reached low levels by the middle of 2008 and then climbed sharply towards the end of the year, when stress in the stock and petroleum markets intensified (Similar to the case of petroleum exporting countries, the stock sector indices of China and the United States experienced large volatility increases during the period 2008-2009 (see the Fig. A.3 and Fig. A.4 in Appendix). These results align with existing literature that consider petroleum importing countries (Filis et al., 2011; Degiannakis et al., 2013; Boldanov et al., 2016; among other).

During the second period of 2010-2013 that incorporates unrest caused by the Arab Spring, time-varying conditional correlations between stock sector indices of China and petroleum remained positive, despite some fluctuations in 2011, and decreased modestly by the end of 2013. On the contrary, dynamic conditional correlations of stock sector indices with petroleum exhibited substantial variations in the case of the United States. The strong positive conditional correlations for all stock sector indices, with the exception of the Energy stock sector index, plummeted, entering negative regions in the first half of 2011 when opposite trends in petroleum prices and stock sector indices were observed. A similar observation with a pronounced decline is reported by Liu et al. (2022), although the values do not enter negative territory. The sharp increase of conditional correlations during the second half of 2011 and swings observed in 2012 could be elucidated by the aforementioned factors, as in the case of petroleum exporting countries. It appears that the relative stability in the petroleum market contributed to the decline of conditional correlations in 2013.

The final period of interest from 2014 to 2018 resulted in the different behaviour of dynamic conditional correlations between petroleum and stock sector indices of petroleum importing countries. The conditional correlations for stock sector indices of the United States increased moderately by the end of 2014, when petroleum prices experienced the initial phase of drops, and bounced back to the low level during the first part of 2015. In the case of China, the country's stock market surge prior to the crash could provide a plausible explanation for the steep decline in conditional correlations over the same period. For the United States, stock sector indices displayed increased conditional correlations with petroleum from mid-2015 to mid-2016. On the other hand, time-varying conditional correlations of Chinese stock sector indices with petroleum strengthened considerably, hitting the highest level during the latter part of 2015 that incorporated the stock market turbulence amid the slowdown in the global economic growth. Fig. A.4 in Appendix depict that stock sector indices of China experienced abnormal spikes in volatilities during the market crash. The second half of the final period exhibited relatively low correlations between petroleum and stock sector indices of petroleum importers. The documented patterns in dynamic correlations exhibit similarities with the findings reported by Liu et al. (2020), Liu et al. (2022) and Zhao & Wang (2022), who focus on aggregate market indices.

Figure 1. Dynamic conditional correlations between petroleum and stock sector indices of petroleum exporting and importing countries from the VAR-DCC-GARCH model.



Overall, the empirical results indicate that the analysis of dynamic conditional correlations between petroleum and stock sector indices reveals abundant and interesting information, thereby suggesting that making investment decisions based on constant conditional correlations would be misleading. The lowest time-varying conditional correlations, despite the hikes in values during the major events, were observed for stock sector indices of the net petroleum exporter (Saudi Arabia) and net petroleum importer (China) with developing markets, which points to potential opportunities for portfolio diversification. This finding is consistent with the studies of Maghyereh et al. (2017) and Boldanov et al. (2016) that consider aggregate market indices for Saudi Arabia and China, respectively. Furthermore, it can be confirmed that conditional correlations vary substantially among stock sector indices of petroleum-exporting and importing countries. Nevertheless, the global events causing uncertainty in both stock and petroleum markets resulted in increased correlations, although with differences in magnitude, regardless of the country's status. Thus, this outcome supports the view that correlations tend to be greater during the worst crisis periods (Sadorsky, 2014a).

6. Portfolio diversification and hedging effectiveness

The obtained empirical findings on shock and volatility transmissions, including dynamic conditional correlations, between stock sector indices of petroleum exporting (Canada and Saudi Arabia) and importing (China and the United States) countries and petroleum necessitate the analysis of their implications for the diversification of investment portfolios and management of risks. The efficient hedging of exposure to swings in petroleum prices is of great interest to market participants. To this end, the present study quantifies optimal portfolio weights and hedge ratios utilising estimates of VAR-BEKK-GARCH, VAR-GARCH, VAR-AGARCH, VAR-DCC-GARCH models for stock sector index and petroleum pairs. Furthermore, the performance of the considered four GARCH specifications in terms of risk reduction is examined.

6.1. Optimal portfolio holdings

The optimal weights of holding a stock sector index and petroleum assets in a portfolio that intends to minimise risks while maintaining the same expected returns are determined following the approach of Kroner & Ng (1998):

$$w_{ij,t} = \frac{h_{jj,t} - h_{ij,t}}{h_{ii,t} - 2h_{ij,t} + h_{jj,t}} \tag{11}$$

where, the weight of the first asset in a one-dollar portfolio comprising a stock sector index and petroleum at time t is denoted as $w_{ij,t}$, $h_{ii,t}$ and $h_{jj,t}$ signify the conditional variances of assets i and j at time t, respectively, and $h_{ij,t}$ refers to the conditional covariance between two assets at time t. Hence, the weight of the second asset in the same portfolio is computed as $1 - w_{ij,t}$. Since a short-selling strategy is not permitted, the following constraints are imposed during the construction process of portfolios:

$$w_{ij,t} = \begin{cases} 0, & if\ w_{ij,t} < 0 \\ w_{ij,t}, & if\ 0 \leq w_{ij,t} \leq 1 \\ 1, & if\ w_{ij,t} > 1 \end{cases} . \tag{12}$$

Table 6 (panels A and B) reports the average daily values of optimal portfolio weights obtained from the estimates of four multivariate GARCH approaches. The optimal weights for portfolios related to each model vary within similar ranges. However, it is worth noting that the results exhibit substantial differences in values across sectors of petroleum exporting and importing countries. For instance, average weights for the Industrials/Brent portfolios obtained from the VAR-DCC-GARCH model are 0.854 for Canada, 0.782 for the United States, 0.595 for Saudi Arabia and 0.546 for China. These figures suggest that in a $1 portfolio, 85.4, 78.2, 59.5 and 54.6 cents should be invested in the Industrials stock sector indices, and the remaining amounts of 14.6, 21.8, 40.5 and 45.4

cents in Brent crude petroleum. The highest weights of stock sector indices from four models were detected in the Consumer Non-Cyclicals/Brent and Financials/Brent portfolios of Canada, and the Consumer Non-Cyclicals/Brent portfolio of the United States. On the other hand, all models indicate that optimal holdings of petroleum exceed stock sector indices only in the Basic Materials/Brent portfolio of Canada and the Energy/Brent portfolio of China. Overall, portfolios should include more stock sector indices than petroleum assets in order to minimise risks while maintaining the same level of returns, which is in line with findings of Arouri et al. (2011b), Arouri et al. (2012), Lin et al. (2014) and Hamma et al. (2021). However, it is interesting to observincludingptimal weights of Brent crude petroleum, regardless of the considered multivariate GARCH approach, are greater in portfolios composed of stock sector indices of the net petroleum exporter and importer with developing markets, namely Saudi Arabia and China, thereby implying that risks associated with petroleum prices are lower for these two countries.

6.2. Optimal hedge ratios

Applying the methodology of Kroner & Sultan (1993), to reduce risks associated with a portfolio consisting of a stock sector index and petroleum, the optimal hedge ratios between two assets are derived as follows:

$$\beta_{ij,t} = \frac{h_{ij,t}}{h_{jj,t}}. \tag{13}$$

where, $\beta_{ij,t}$ represents a risk-minimising optimal hedge ratio at time t. Specifically, a $1 long position taken in one asset at time t must be hedged by a short position equivalent to a dollar amount of $\beta_{ij,t}$ in the other asset. Thus, a hedge is defined as efficient or inexpensive, if the associated hedge ratios have low values (Hammoudeh et al., 2010; Maghyereh et al., 2017). It is however important to note that negative optimal hedge ratios can also occur over the sample period and, therefore, the obtained values have the reverse interpretation.

The average daily figures of optimal hedge ratios generated from the estimates of four multivariate GARCH approaches are also reported in Table 6 (panels A and B). The findings show that optimal hedge ratios differ considerably across models and sectors of petroleum exporting and importing countries. For example, a glance at the portfolios with Brent crude petroleum and the Industrial stock sector index suggests that the largest hedge ratios were provided by the VAR-DCC-GARCH model that equal to 0.639 for Canada, 0.353 for the United States, 0.190 for Saudi Arabia and 0.104 for China. These values imply that a long position of $1 in Brent crude petroleum should be hedged with short positions of 63.9, 35.3, 19.0 and 10.4 cents in the Industrial stock sector indices of four countries. One can observe that the most and least efficient strategies to hedge risks associated with petroleum prices are to take short positions in the Financials stock sector indices of China and Canada, respectively, as shown by corresponding figures from four models. Among stock sector indices of all countries, the lowest hedging costs are between the Consumer Non-Cyclicals stock sector indices of the United States and China and Brent crude petroleum, where an investment of $1 in the stock sector index should be shorted by less than 10 cents in petroleum. The four multivariate GARCH models produced the smallest optimal hedge ratios in the case of Saudi Arabia (net exporter) and China (net importer), thereby suggesting that stock sector indices of these countries generally offer lower costs related to hedging petroleum risk exposure. This outcome is consistent with the studies of Arouri et al. (2011c), Maghyereh et al. (2017) and Yousaf & Hassan (2019) that consider aggregate market indices for Saudi Arabia and China. Overall, the highest values of optimal hedge ratios were mostly obtained from the VAR-DCC-GARCH model, which indicates that more assets, that is, a stock sector index or petroleum depending on the long positions, should be shorted to minimise risks of portfolios for investors.

Table 6. Optimal portfolio weights and hedge ratios for petroleum-exporting and importing countries.

Panel A: Exporters	Canada								Saudi Arabia							
Portfolio	VAR-BEKK-GARCH		VAR-GARCH		VAR-AGARCH		VAR-DCC-GARCH		VAR-BEKK-GARCH		VAR-GARCH		VAR-AGARCH		VAR-DCC-GARCH	
	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t
Basic Materials/Brent	0.472	0.354	0.470	0.361	0.473	0.360	0.465	0.382	0.678	0.118	0.671	0.126	0.670	0.122	0.681	0.116
Brent/Basic Materials	0.528	0.322	0.530	0.326	0.527	0.326	0.535	0.341	0.322	0.267	0.329	0.242	0.330	0.231	0.319	0.278
Consumer Cyclicals/Brent	0.844	0.178	0.845	0.178	0.841	0.176	0.858	0.193	0.644	0.070	0.636	0.089	0.633	0.083	0.647	0.074
Brent/Consumer Cyclicals	0.156	0.571	0.155	0.590	0.159	0.583	0.142	0.620	0.356	0.197	0.364	0.151	0.367	0.137	0.353	0.188
Consumer Non-Cyclicals/Brent	0.870	0.137	0.872	0.142	0.868	0.140	0.884	0.153	0.581	0.072	0.569	0.103	0.568	0.098	0.581	0.074
Brent/Consumer Non-Cyclicals	0.130	0.573	0.128	0.570	0.132	0.559	0.116	0.612	0.419	0.166	0.431	0.130	0.432	0.120	0.419	0.162
Energy/Brent	0.583	0.466	0.583	0.462	0.591	0.460	0.588	0.475	0.589	0.110	0.577	0.129	0.576	0.124	0.589	0.110
Brent/Energy	0.417	0.545	0.417	0.550	0.409	0.555	0.412	0.563	0.411	0.203	0.423	0.171	0.424	0.162	0.411	0.204
Financials/Brent	0.872	0.191	0.883	0.182	0.882	0.179	0.892	0.201	0.624	0.080	0.618	0.088	0.622	0.084	0.626	0.099
Brent/Financials	0.128	0.678	0.117	0.709	0.118	0.699	0.108	0.736	0.376	0.159	0.382	0.139	0.378	0.133	0.374	0.184
Industrials/Brent	0.831	0.202	0.840	0.199	0.839	0.195	0.854	0.218	0.594	0.100	0.585	0.119	0.584	0.114	0.595	0.106
Brent/Industrials	0.169	0.590	0.160	0.610	0.161	0.603	0.146	0.639	0.406	0.189	0.415	0.160	0.416	0.150	0.405	0.190
Panel B: Importers	China								United States							
Portfolio	VAR-BEKK-GARCH		VAR-GARCH		VAR-AGARCH		VAR-DCC-GARCH		VAR-BEKK-GARCH		VAR-GARCH		VAR-AGARCH		VAR-DCC-GARCH	
	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t	w_t	β_t
Basic Materials/Brent	0.500	0.105	0.496	0.111	0.492	0.112	0.495	0.117	0.715	0.171	0.717	0.165	0.718	0.161	0.720	0.185
Brent/Basic Materials	0.500	0.117	0.504	0.110	0.508	0.109	0.505	0.115	0.285	0.352	0.283	0.365	0.282	0.359	0.280	0.386
Consumer Cyclicals/Brent	0.546	0.075	0.542	0.080	0.538	0.082	0.542	0.084	0.745	0.094	0.745	0.081	0.744	0.077	0.752	0.101
Brent/Consumer Cyclicals	0.454	0.106	0.458	0.097	0.462	0.097	0.458	0.103	0.255	0.218	0.255	0.232	0.256	0.223	0.248	0.258
Consumer Non-Cyclicals/Brent	0.566	0.063	0.562	0.065	0.558	0.066	0.563	0.072	0.865	0.050	0.864	0.039	0.860	0.035	0.876	0.055
Brent/Consumer Non-Cyclicals	0.434	0.096	0.438	0.085	0.442	0.085	0.437	0.098	0.135	0.211	0.136	0.239	0.140	0.214	0.124	0.279
Energy/Brent	0.474	0.132	0.471	0.135	0.467	0.137	0.471	0.142	0.583	0.395	0.574	0.401	0.577	0.396	0.581	0.402
Brent/Energy	0.526	0.126	0.529	0.122	0.533	0.121	0.529	0.132	0.417	0.471	0.426	0.479	0.423	0.475	0.419	0.486
Financials/Brent	0.524	0.069	0.519	0.074	0.516	0.074	0.520	0.081	0.712	0.121	0.713	0.110	0.712	0.104	0.713	0.119
Brent/Financials	0.476	0.087	0.481	0.082	0.484	0.081	0.480	0.098	0.288	0.258	0.287	0.282	0.288	0.269	0.287	0.272
Industrials/Brent	0.551	0.077	0.546	0.078	0.542	0.079	0.546	0.085	0.775	0.117	0.776	0.106	0.775	0.103	0.782	0.125
Brent/Industrials	0.449	0.104	0.454	0.095	0.458	0.095	0.454	0.104	0.225	0.312	0.224	0.331	0.225	0.329	0.218	0.353

Notes: w_t and β_t refer to average weights and hedge ratios (long/short), respectively, of assets in the portfolio consisting of a stock sector index and petroleum, which are computed using the conditional variance and covariance estimates of four multivariate GARCH approaches.

6.3. Hedging performance analysis

Given that the results from the previous sub-sections support the view that the inclusion of a petroleum asset in a portfolio of stock sector indices improves its risk-adjusted performance (Arouri et al., 2012), it is natural to wonder about the effectiveness of hedging strategies. In order to analyse the performance of optimal hedge ratios obtained from estimates of four multivariate GARCH models, the study first constructs two portfolios: (i) an unhedged portfolio that only includes stock sector index or petroleum assets; and (ii) a hedged portfolio composed of both stock sector index and petroleum assets. The return on a hedged portfolio is computed as $r_{h,t} = r_{i,t} - \beta_t r_{j,t}$, where β_t refers to the daily optimal hedge ratios, $r_{i,t}$ and $r_{j,t}$ represent the daily returns of holding assets i and j at time t, respectively. Following the studies of Ku et al. (2007) and Chang et al. (2011), which suggest that a more precise conditional volatility approach is also expected to outperform in terms of risk elimination, the present work employs a hedging effective (HE) index. This indicator measures the percentage decrease in the variance of a hedged portfolio as opposed to an unhedged portfolio and is defined as follows:

$$HE = \left(\frac{var_{unhedged} - var_{hedged}}{var_{unhedged}}\right) \tag{14}$$

where, $var_{unhedged}$ refers to the variance of unhedged portfolios' returns, that is, the variance of stock sector indices or petroleum returns, and var_{hedged} represents the variance of hedged portfolios' returns comprising stock sector index and petroleum assets, that is, the variance of $r_{h,t}$. The greater HE values indicate that a hedging method is more effective in terms of reducing variances of portfolios. Hence, a multivariate GARCH approach, which produces the highest values of HE, is considered to be superior for constructing hedging strategies.

Table 7 (panels A and B) presents the variances of hedged portfolios and hedging effectiveness figures produced from the considered multivariate GARCH models. The findings indicate that the hedging strategies including stock sector index and petroleum assets lead to the effective reduction in the risk of portfolios that is represented by the variance. The comparison of four multivariate GARCH models reveals that the VAR-DCC-GARCH specification, which incorporates time-varying correlations between variables, performs better in terms of decreasing the variances of portfolios as shown by the highest HE values in most cases. The reduction of all portfolios' variances ranges from 13.528% (Brent/Consumer Non-Cyclicals) to 31.647% (Energy/Brent) for Canada, from 0.702% (Consumer Non-Cyclicals/Brent) to 5.979% (Brent/Basic Materials) for Saudi Arabia, from 0.859% (Brent/Consumer Non-Cyclicals) to 2.371% (Energy/Brent) for China, and from 3.018% (Brent/Consumer Non-Cyclicals) to 19.289% (Brent/Energy) for the United States. On the contrary, the VAR-BEKK-GARCH specification provides the lowest HE figures, which suggests that this specification is the least efficient in minimising the portfolios' variances, particularly those that include stock sector indices of Saudi Arabia and China. Chang et al. (2011) also documented the worst performance of the BEKK model in their study, which focuses on analysing the effectiveness of optimal hedge ratios between the major crude petroleum, Brent and WTI, spot and futures markets. The risk reduction differs substantially across sectors of petroleum-exporting and importing countries. Specifically, the VAR-DCC-GARCH specification resulted in the largest HE values for portfolios composed of the Energy stock sector indices and Brent crude petroleum in the case of Canada, China, and the United States, and the Basic Materials stock sector index and Brent crude petroleum in the case of Saudi Arabia. The lowest HE values were generated for portfolios of all countries comprising the Consumer Non-Cyclicals stock sector indices and Brent crude petroleum. It can be noted that the greatest variance reduction, irrespective of the multivariate GARCH models, and therefore, the better hedging effectiveness, is detected for portfolios involving stock sector indices of Canada and the United States, which appear to be more exposed to petroleum price risks. However, the low HE figures for portfolios of Saudi Arabia and China do not necessarily imply that the hedging strategies are questionable as long as the variance reductions are sufficient to compensate for transaction costs. Overall, the results point to the outperformance of optimally hedged portfolios compared to conventional portfolios composed of stock sector indices or petroleum assets only, thereby emphasising the importance of diversification, which reinforces the conclusions drawn by Arouri et al. (2012).

Table 7. Performance of optimal hedge ratios for portfolios of petroleum-exporting and importing countries.

Panel A: Exporters	Canada								Saudi Arabia							
Portfolio	VAR-BEKK-GARCH		VAR-GARCH		VAR-AGARCH		VAR-DCC-GARCH		VAR-BEKK-GARCH		VAR-GARCH		VAR-AGARCH		VAR-DCC-GARCH	
	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)
Basic Materials/Brent	4.365	16.158	4.408	15.317	4.393	15.616	4.335	16.721	3.680	1.393	3.628	2.778	3.643	2.384	3.611	3.247
Brent/Basic Materials	3.913	13.881	3.920	13.726	3.918	13.762	3.879	14.630	4.331	4.681	4.339	4.494	4.336	4.559	4.272	5.979
Consumer Cyclicals/Brent	1.159	17.212	1.191	14.931	1.192	14.834	1.156	17.403	4.374	0.027	4.334	0.942	4.342	0.755	4.313	1.435
Brent/Consumer Cyclicals	3.892	14.342	3.929	13.535	3.926	13.600	3.864	14.957	4.449	2.071	4.436	2.368	4.436	2.362	4.370	3.819
Consumer Non-Cyclicals/Brent	0.919	14.542	0.949	11.723	0.951	11.565	0.916	14.814	5.724	-0.454	5.685	0.227	5.697	0.017	5.658	0.702
Brent/Consumer Non-Cyclicals	3.944	13.191	4.021	11.494	4.016	11.606	3.929	13.528	4.455	1.936	4.442	2.222	4.442	2.217	4.382	3.551
Energy/Brent	2.967	31.066	2.961	31.192	2.960	31.219	2.942	31.647	5.017	0.687	4.988	1.262	5.001	1.010	4.944	2.148
Brent/Energy	3.227	28.969	3.222	29.083	3.213	29.295	3.185	29.906	4.392	3.331	4.391	3.344	4.390	3.377	4.333	4.631
Financials/Brent	1.319	19.925	1.344	18.415	1.347	18.200	1.302	20.929	3.319	1.794	3.315	1.923	3.328	1.534	3.285	2.805
Brent/Financials	3.747	17.538	3.814	16.050	3.808	16.196	3.718	18.175	4.439	2.299	4.437	2.348	4.431	2.477	4.384	3.498
Industrials/Brent	1.391	20.414	1.435	17.878	1.432	18.065	1.391	20.403	5.096	0.968	5.086	1.162	5.102	0.836	5.032	2.202
Brent/Industrials	3.797	16.437	3.838	15.526	3.834	15.611	3.768	17.068	4.400	3.156	4.407	2.997	4.405	3.038	4.340	4.462
Panel B: Importers	China								United States							
Portfolio	VAR-BEKK-GARCH		VAR-GARCH		VAR-AGARCH		VAR-DCC-GARCH		VAR-BEKK-GARCH		VAR-GARCH		VAR-AGARCH		VAR-DCC-GARCH	
	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)	σ^2 (%)	HE (%)
Basic Materials/Brent	4.902	0.857	4.874	1.411	4.875	1.392	4.859	1.718	2.305	7.217	2.289	7.881	2.292	7.768	2.267	8.769
Brent/Basic Materials	4.699	0.416	4.648	1.505	4.646	1.556	4.643	1.608	4.164	7.904	4.194	7.241	4.187	7.382	4.133	8.591
Consumer Cyclicals/Brent	4.085	0.336	4.062	0.898	4.064	0.844	4.048	1.231	2.039	5.503	2.084	3.450	2.086	3.350	2.019	6.466
Brent/Consumer Cyclicals	4.736	-0.350	4.674	0.950	4.673	0.978	4.674	0.952	4.305	4.785	4.394	2.807	4.391	2.862	4.276	5.417
Consumer Non-Cyclicals/Brent	3.782	-0.264	3.748	0.631	3.750	0.589	3.734	1.015	0.797	3.680	0.811	2.106	0.811	2.010	0.796	3.883
Brent/Consumer Non-Cyclicals	4.737	-0.382	4.683	0.763	4.681	0.800	4.679	0.859	4.436	1.878	4.457	1.415	4.459	1.380	4.384	3.018
Energy/Brent	5.116	1.768	5.100	2.075	5.102	2.039	5.085	2.371	3.711	15.326	3.579	18.345	3.585	18.204	3.586	18.174
Brent/Energy	4.644	1.596	4.618	2.139	4.615	2.214	4.609	2.342	3.716	17.803	3.672	18.782	3.669	18.851	3.649	19.289
Financials/Brent	4.319	0.031	4.293	0.653	4.294	0.611	4.274	1.092	3.850	-0.374	3.755	2.098	3.761	1.931	3.719	3.043
Brent/Financials	4.718	0.033	4.677	0.882	4.675	0.929	4.674	0.960	4.320	4.437	4.387	2.968	4.386	2.985	4.283	5.266
Industrials/Brent	3.961	1.048	3.964	0.975	3.965	0.964	3.929	1.852	1.670	7.171	1.699	5.570	1.701	5.445	1.657	7.900
Brent/Industrials	4.711	0.171	4.675	0.944	4.673	0.967	4.668	1.091	4.243	6.142	4.317	4.521	4.309	4.695	4.214	6.786

Notes: HE denotes hedging effectiveness. σ^2 refers to the hedged portfolios' variances, while the square of standard deviations reported in Tables 2 and 3 represents the variances of unhedged portfolios composed of stock sector indices or Brent crude petroleum only. The figures highlighted in bold point to the multivariate GARCH model that produced the lowest variance and the greatest reduction of variance for the hedged portfolio.

6.4. Time-varying optimal portfolio weights and hedge ratios

The Figs. 2 and 3 depict the time-varying optimal portfolio weights and hedge ratios computed based on variance and covariance estimates of the VAR-DCC-GARCH specification considering its superiority in terms of hedging effectiveness. The dynamic figures provide important insights to better understand risks throughout the study period, although the importance of average portfolio weights and hedge ratios should not be underestimated.

Starting first with optimal portfolio weights, it can be observed that the values exhibit significant fluctuations, pointing to the necessity for active management of portfolios, particularly during the global events. The time-varying weights of portfolios comprising stock sector indices of Saudi Arabia and China mostly move in tandem. On the other hand, the directions of portfolio weights differ across stock sector indices of Canada and the United States. The findings indicate that the major events, accompanied by considerable declines in petroleum prices, led to the greater optimal holdings of stock sector indices in portfolios of all countries. Conversely, the uncertainty in stock markets associated, for instance, with the global financial crisis, Eurozone debt crisis, downgrade of the Unites States credit rating, collapse of Saudi and Chinese stock markets, resulted in the increased optimal weights of the petroleum asset. Furthermore, the period of Arab Spring, which witnessed the upward trend in petroleum prices, show that optimal weights of petroleum remained high in portfolios of Saudi Arabia and China, but low in portfolios of Canada and the United States. Interestingly, during the peak periods of instabilities in the stock and petroleum markets, the weights related to all portfolios of Canada and the United States, with the exception of the Energy/Brent portfolio, suggest a 100% investment in stock sector indices or petroleum assets in order to minimise risks. These observations align with previous studies (Liu et al., 2023; Ramesh et al., 2025; among others), which also report that portfolio holdings exhibit significant fluctuations during periods of market turbulence.

Turning to the optimal hedge ratios, one can note that the figures are rather volatile, reaching the top and bottom levels over the turbulent periods associated with the major events. The dynamic hedge ratios vary across sectors of petroleum exporting and importing countries. The lower values are obtained for portfolios of all countries with long and short positions taken in stock sector indices and petroleum, respectively. At the beginning of the period 2005-2009, the hedge ratios are relatively steady for Canada, China and the United States. In the case of Saudi Arabia, they turn negative during the country’s stock market collapse, pointing to a short position in the first asset and a long position in the second asset. In the first half of 2008, the global financial crisis led to negative hedge ratios. However, the recession caused substantially high values from the mid-2008 and, hence, increased costs of hedging, particularly for portfolios of Canada and the United States. During the second period from 2010 to 2013, which is associated with the Arab Spring, all hedge ratios experienced drops by the mid-2011, taking the negative forms in the case of Saudi Arabia and the United States. The figures surged in the second half of 2011, which could be attributed to the European debt crisis and downgrade of the credit rating of the United States, and gradually decreased by the end of 2013. The final period of 2014-2018, which incorporates the geopolitical tensions and imbalances in the petroleum market, indicates that all optimal hedge ratios, particularly related to Petroleum/Sector portfolios, fluctuate dramatically within the high ranges from the end of 2014 until the end of 2016. Furthermore, it is worth mentioning that the hedging costs for China's sector/petroleum portfolios spiked during the stock market meltdown. The optimal hedge ratios stabilised over the period 2017-2018. The findings are consistent with the studies conducted by Maghyereh et al. (2017) and Antonakakis et al. (2018), which also report significant increases in hedge ratios during periods of crisis. Overall, the time-varying optimal hedge ratios remain small, despite the hikes in turmoil times, for both Sector/Petroleum and Petroleum/Sector portfolios involving stock sector indices of Saudi Arabia and China.

Figure 2. Time-varying optimal portfolio weights obtained from the VAR-DCC-GARCH model for petroleum-exporting and importing countries.

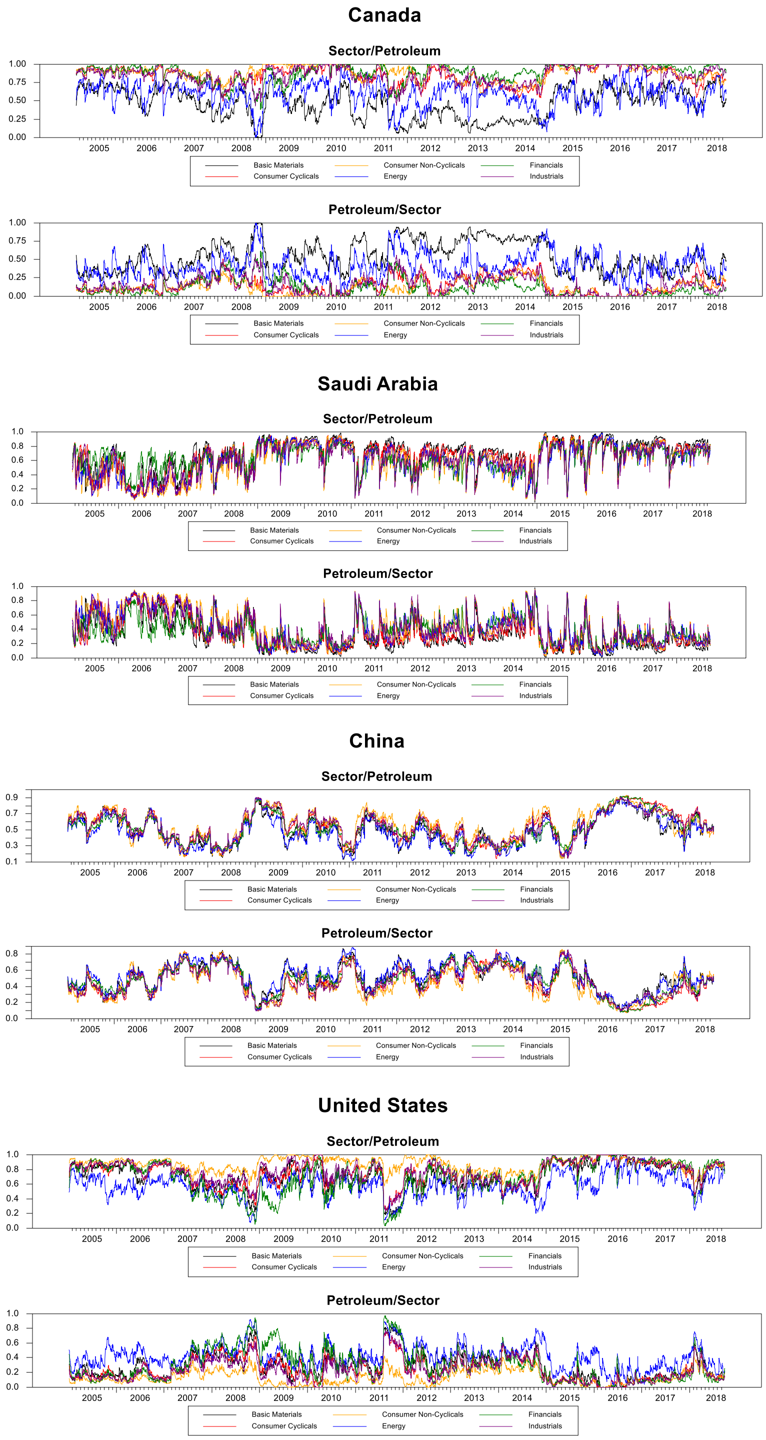
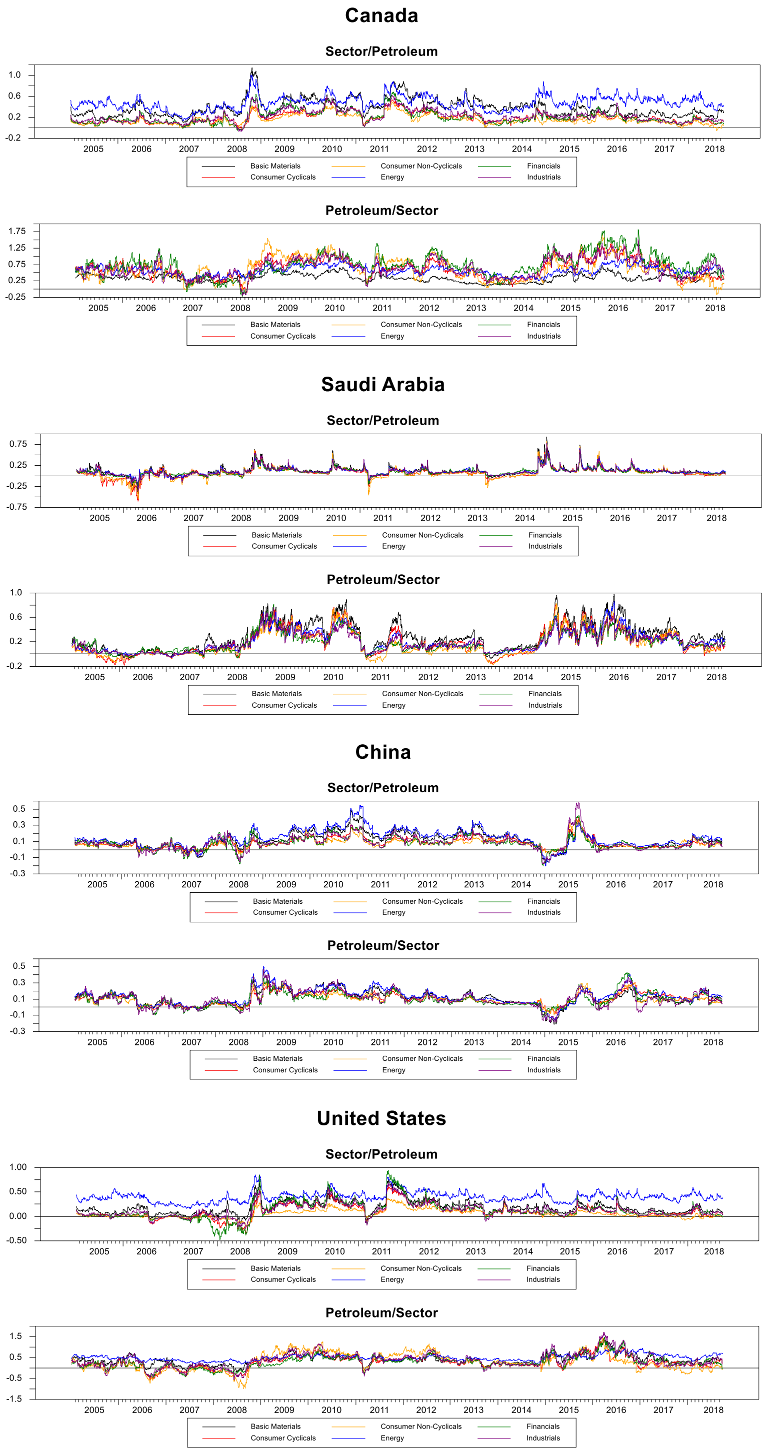


Figure 3. Time-varying optimal hedge ratios obtained from the VAR-DCC-GARCH model for petroleum-exporting and importing countries.



7. Research implications

The findings of the present study have notable implications for market participants, policymakers, and the research community focused on investigating linkages between petroleum stock markets, as well as volatility modelling. The comparative evaluation of four multivariate GARCH specifications, such as VAR-BEKK-GARCH, VAR-GARCH, VAR-AGARCH, and VAR-DCC-GARCH, provides detailed insights into dynamic interactions between petroleum prices and equity sections across petroleum-exporting and importing countries with different levels of market development. Furthermore, the outputs have practical applications in portfolio construction and risk management, and also contribute to a broader understanding of the financial market interdependence.

The observation of more pronounced volatility spillovers in the case of Canada and the United States, which mostly take place from stock sector indices to petroleum, suggests that stock market trends in petroleum-exporting advanced economies could act as precursors to probable changes in the global petroleum market. Consequently, a thorough analysis of equity market trends, particularly during times of increased uncertainty, could provide valuable insights for both investors and policymakers seeking to anticipate fluctuations in the petroleum price. Furthermore, the detected asymmetries in volatility responses, where negative shocks exert a greater influence than positive counterparts, emphasise the importance of managing downside risk. For risk-averse investors, this observation reinforces the necessity of incorporating models into their analysis that explicitly capture such dynamics in order to avoid underestimation of risk exposure during market downturns.

The higher optimal holdings of petroleum and the lower associated hedging costs in portfolios involving stock sector indices of Saudi Arabia and China, where limited volatility linkages and the presence of foreign ownership restrictions exist, imply that petroleum may function as a more effective hedging tool in developing or less integrated markets. This finding is significant for investors in emerging economies, as it questions the traditional belief that developed markets consistently provide better diversification benefits. Therefore, portfolio construction and hedging strategies should be customised based on country- and sector-specific volatility dynamics, rather than relying on standardised approaches across different markets.

Lastly, the pronounced fluctuations in conditional correlations, portfolio weights, and hedge ratios emphasise the need for adaptive portfolio rebalancing and hedging mechanisms that can effectively respond to shifting market conditions, particularly during turbulent periods. Additionally, the greater effectiveness in reducing portfolio variance achieved by the VAR-DCC-GARCH specification, albeit not substantial, points to a broader insight postulating that the choice of the model matters. Relying on a single specification in modelling conditional volatilities may overlook alternative dynamics that could improve hedging efficiency.

8. Conclusion

The appropriate modelling of volatility dynamics between petroleum and stock markets is essential for the efficient optimisation of portfolios and hedging of energy risks. Despite the fact that multivariate GARCH approaches have been widely applied in this matter, their estimation employing large datasets is a challenging task due to the trade-off between feasibility and generality (Basher & Sadorsky, 2016). For instance, some of the GARCH specifications, such as CCC or DCC, although they reduce the number of free parameters to simplify the estimation process, do not capture volatility spillover effects between variables, which is of current interest given the enhanced level of market integration. The present study applies VAR-BEKK-GARCH, VAR-GARCH, VAR-AGARCH, and VAR-DCC-GARCH models to investigate return and volatility transmissions between Brent crude petroleum spot prices and stock sector indices of two net petroleum exporters (Canada and Saudi Arabia) and two net petroleum importers (China and the United States). The unique methodology of Bagirov and Mateus (2022) is adopted to manually construct stock sector indices utilising the sample of 1,658 stocks listed in the Basic Materials, Consumer Cyclicals, Consumer Non-Cyclicals, Energy, Financials, and Industrials sectors from January 03, 2005, until September 28, 2018. The four multivariate GARCH specifications considered allow for the measurement of bidirectional volatility interactions, which makes them attractive. The conditional variance and covariance estimates are used to obtain optimal portfolio holdings and hedge ratios for stock sector index and petroleum pairs. Furthermore, the ability of hedging strategies associated with each model to reduce portfolio risks is comparatively analysed by applying the hedging effective index.

The empirical findings demonstrate the existence of shock and volatility spillovers between the petroleum and stock sector indices of the studied countries. It should be stressed that the magnitude and course of documented interactions differ across sectors of petroleum-exporting and importing countries, supporting the rationality of the sector-level analysis. The cross effects are more evident in the case of Canada and the United States. For Saudi Arabia and China, past shocks or innovations and volatilities appear to be more important in making predictions of future volatility levels. The asymmetric terms related to petroleum and stock sector indices of Canada, Saudi Arabia, and the United States, with the exception of China, show that negative shocks tend to increase conditional volatilities more than positive shocks of the same extent. The VAR-AGARCH and VAR-DCC-GARCH specifications are found to fit the dataset better. The dynamic conditional correlations between petroleum and stock sector indices of petroleum exporters and importers display heterogeneous behaviour and considerable fluctuations, particularly during the major events that caused uncertainty in the stock and petroleum markets. The low values are reported for stock sector indices of Saudi Arabia and China.

The average optimal portfolio weights and hedge ratios differ considerably across the four countries. The results show that constructed portfolios should generally include more stock sector indices than petroleum assets. Nevertheless, optimal holdings of petroleum remain higher in portfolios comprising stock sector indices of Saudi Arabia and China, as well as the net petroleum exporter and importer with developing markets, where limited evidence of volatility linkages is detected. The examination of hedge ratios reveals that stock sector indices of these two countries, regardless of the models, provide lower costs for hedging petroleum price risks. In addition, the comparative analysis of hedging strategies associated with four multivariate GARCH approaches suggests that the VAR-DCC-GARCH specification, which captures correlation dynamics between variables in the system, produces better outputs and, hence, is more advantageous in terms of minimizing variances of portfolios. Overall, the optimally hedged portfolios outperform their traditional counterparts involving stock sector indices or petroleum assets only, which emphasises the crucial role of diversification. The time-varying portfolio weights and hedge ratios computed based on estimates of the preferred model also display substantial variabilities from one sector to another and tend to be sensitive during turbulent periods in markets.

The empirical frameworks utilised by the present study provide a valuable contribution by uncovering heterogeneous volatility interactions between petroleum and stock sector indices across petroleum-exporting and importing countries, comprehension of which is essential in enhancing strategies of managing risks posed by petroleum price swings. Future research may build upon this work by incorporating higher-frequency data and accounting for structural breaks. Additionally, a comparative analysis between traditional multivariate GARCH models and emerging approaches based on machine learning could facilitate improvement of hedging strategies and forecasting accuracy.

Author contributions: Bagirov Miramir: Writing – original draft, Writing – review & editing, Methodology, Formal analysis, Software, Data curation, Conceptualization. **Mateus Cesario:** Writing – original draft, Writing – review & editing, Methodology, Formal analysis, Software, Data curation, Conceptualization.

Declaration of competing interest: None.

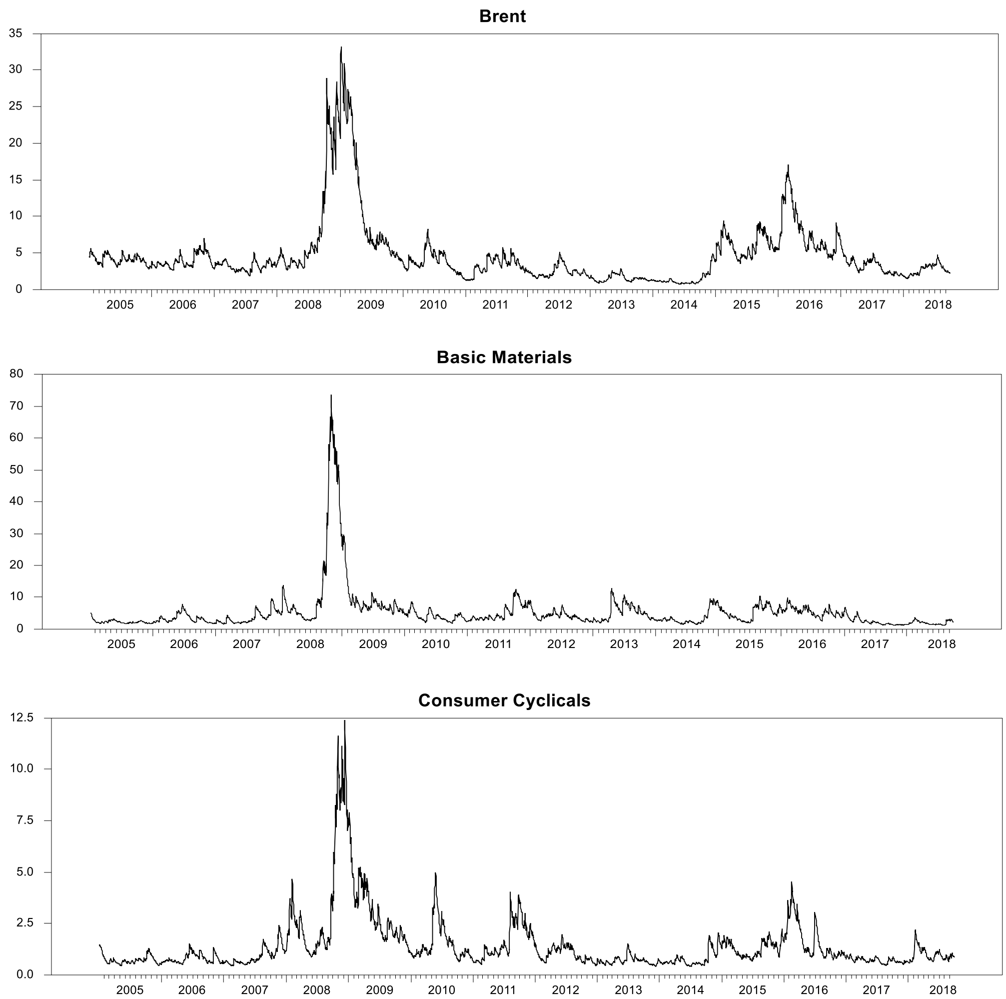
Data availability: The authors do not have permission to share data.

Appendix A

Table A.1: Automatic selection of lag length for the mean equation based on the Schwarz Bayesian Criterion (SBC).

Sector	Canada	China	Saudi Arabia	United States
	SBC Lag	SBC Lag	SBC Lag	SBC Lag
Basic Materials	1	1	1	1
Consumer Cyclicals	1	0	1	1
Consumer Non-Cyclicals	0	0	1	1
Energy	1	1	1	1
Financials	1	0	1	1
Industrials	1	0	1	1

Figure A.1. The time-varying conditional variances of Brent and stock sector indices of Canada obtained from the VAR-DCC-GARCH model.



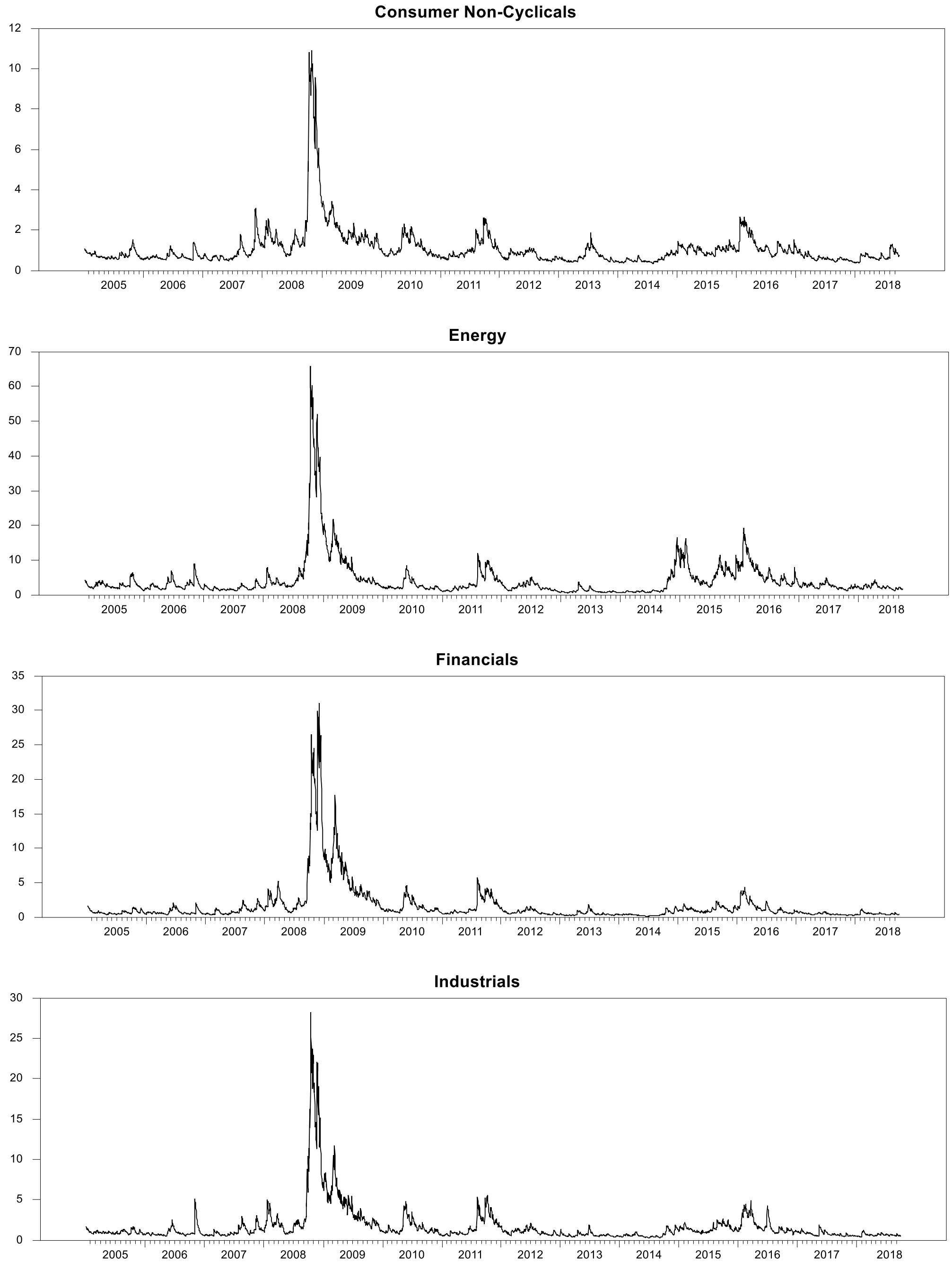
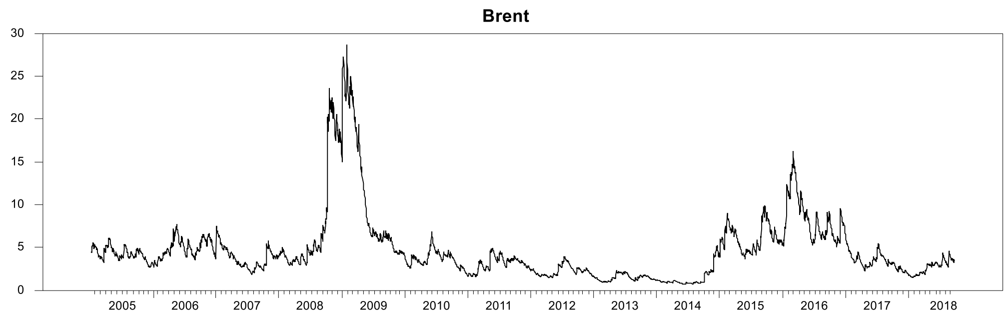
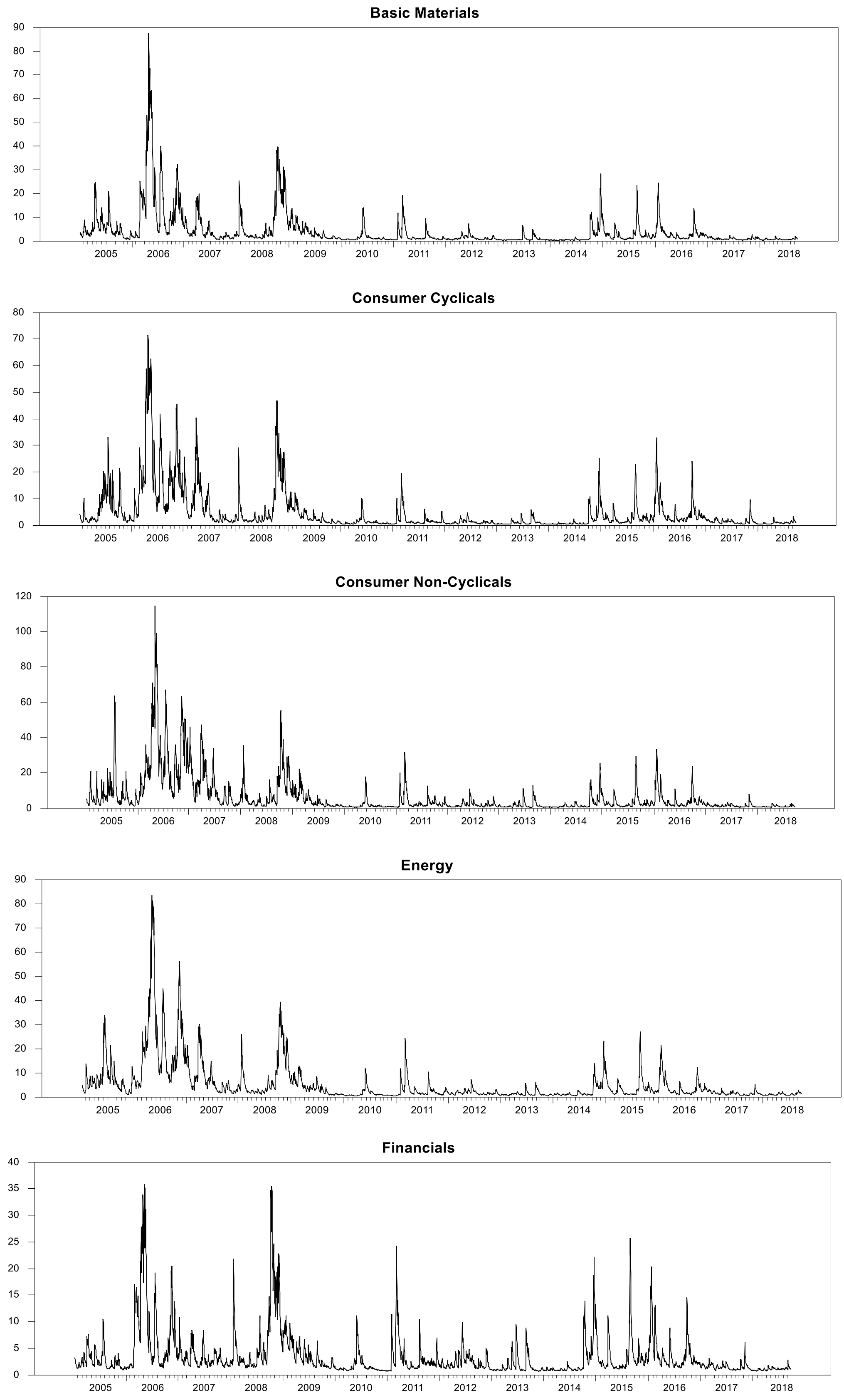


Figure. A.2. The time-varying conditional variances of Brent and stock sector indices of Saudi Arabia obtained from the VAR-DCC-GARCH model.




Basic Materials
Consumer Cyclicals
Consumer Non-Cyclicals
Energy
Financials
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2016
2017
2018

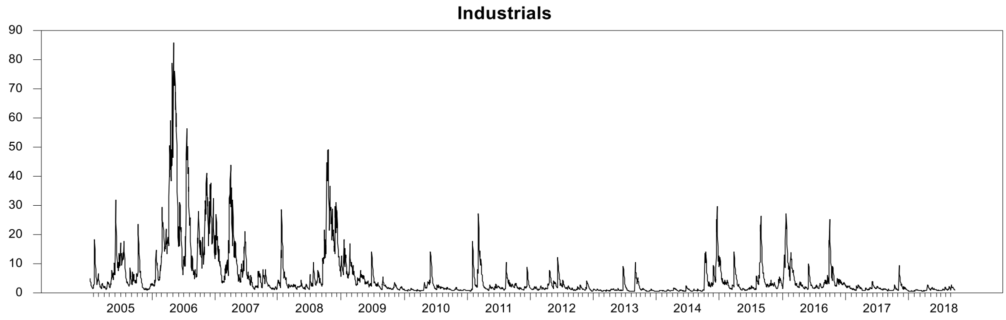
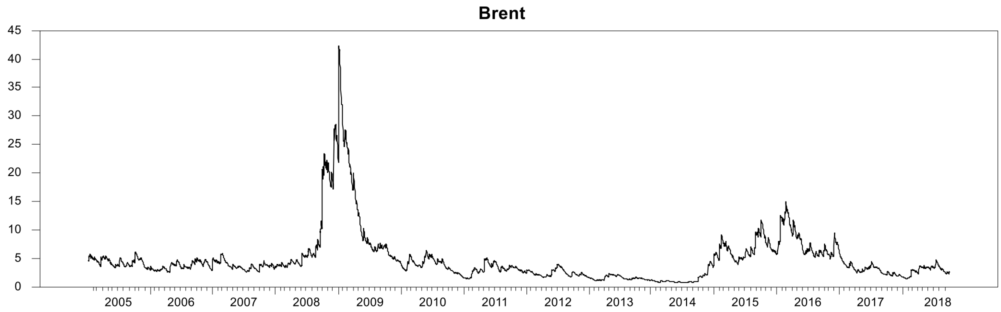
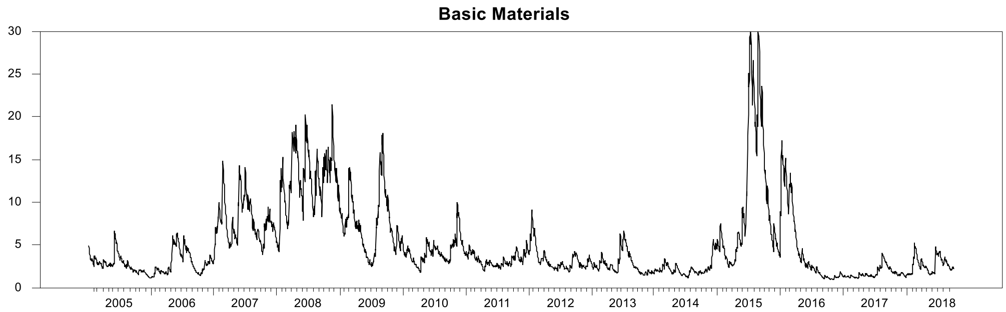
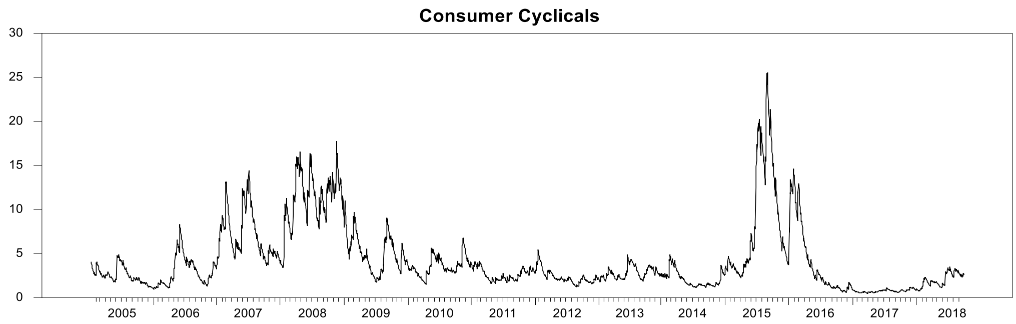
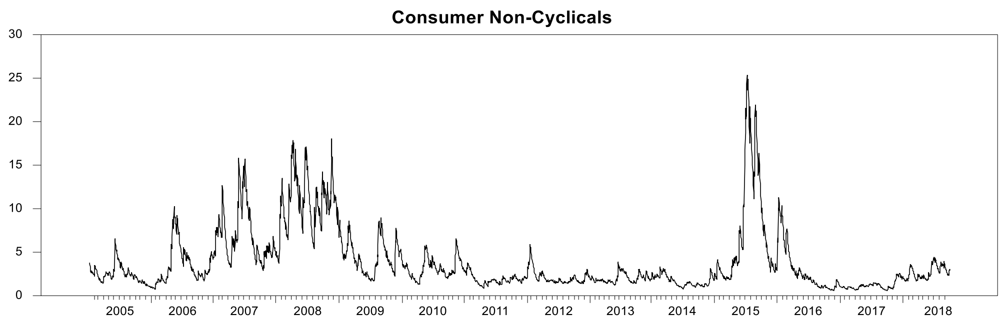


Figure A.3. The time-varying conditional variances of Brent and stock sector indices of China obtained from the VAR-DCC-GARCH model.









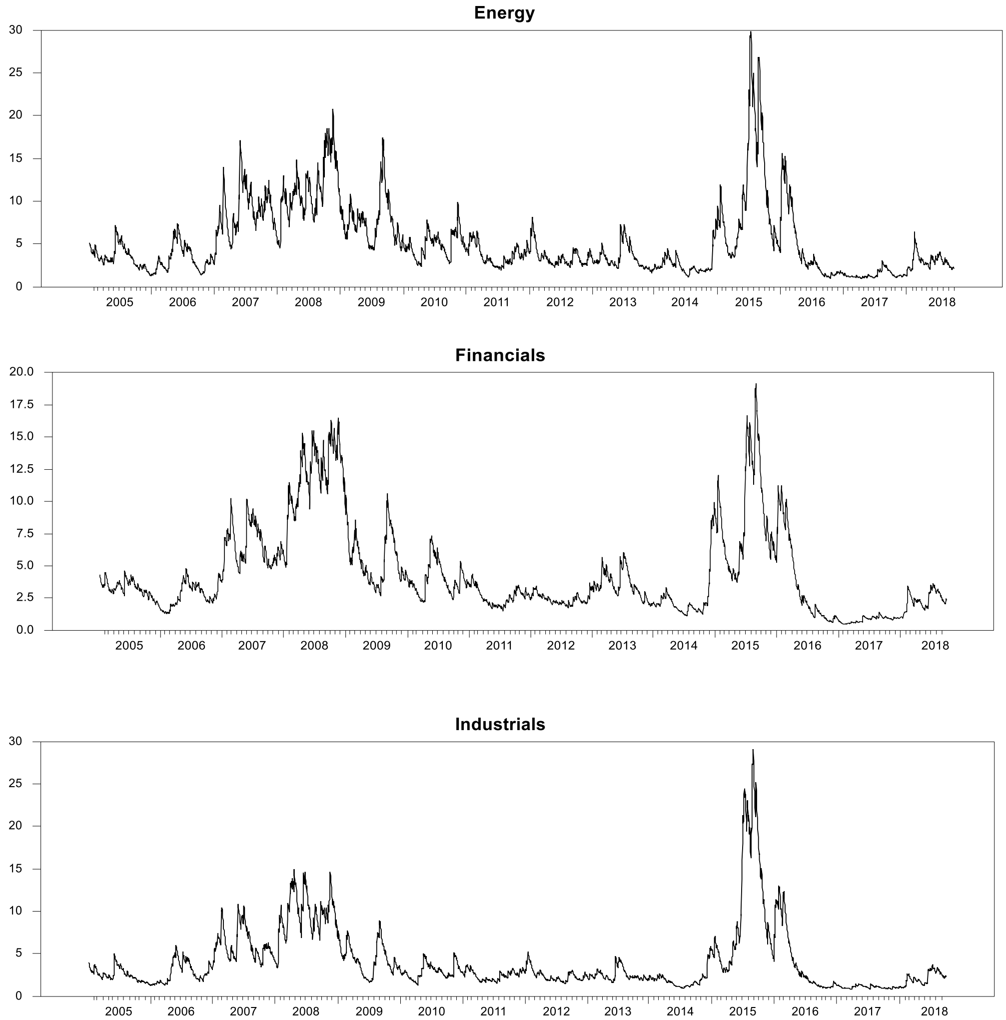
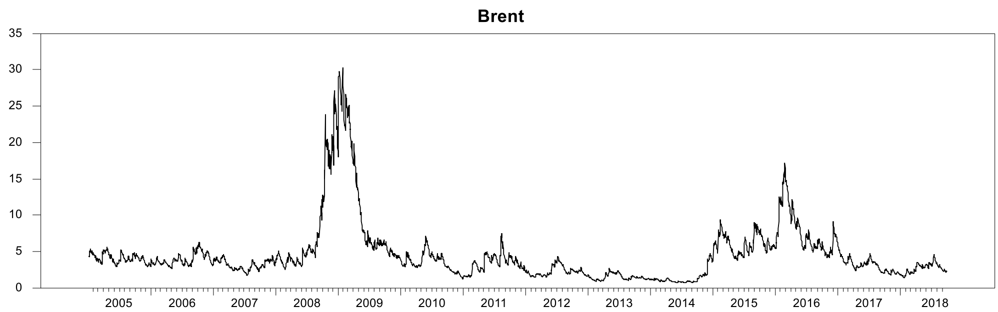
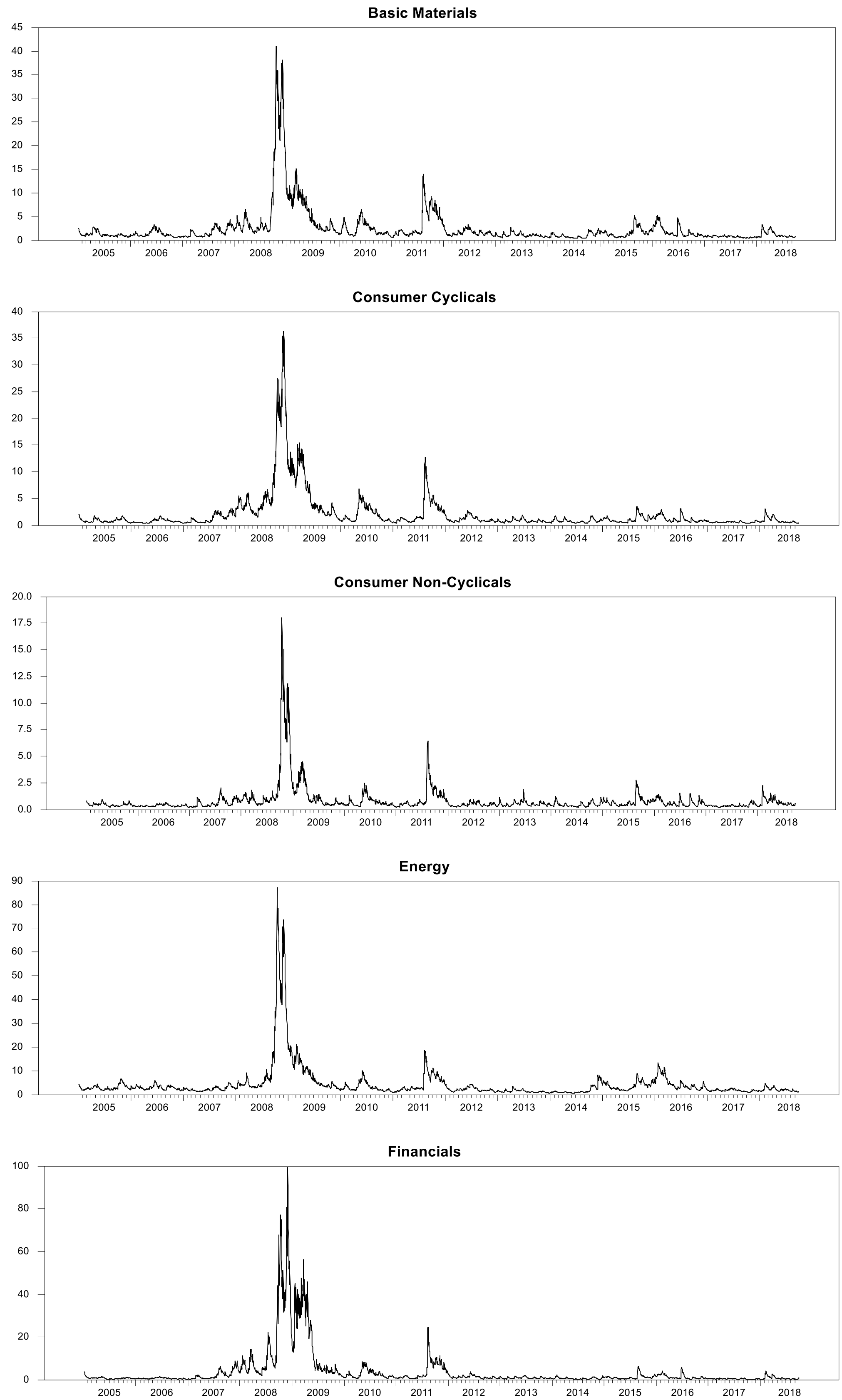


Figure A.4. The time-varying conditional variances of Brent and stock sector indices of the United States obtained from the VAR-DCC-GARCH model.




Basic Materials
45
40
35
30
25
20
15
10
5
0
2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018
Consumer Cyclicals
40
35
30
25
20
15
10
5
0
2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018
Consumer Non-Cyclicals
20.0
17.5
15.0
12.5
10.0
7.5
5.0
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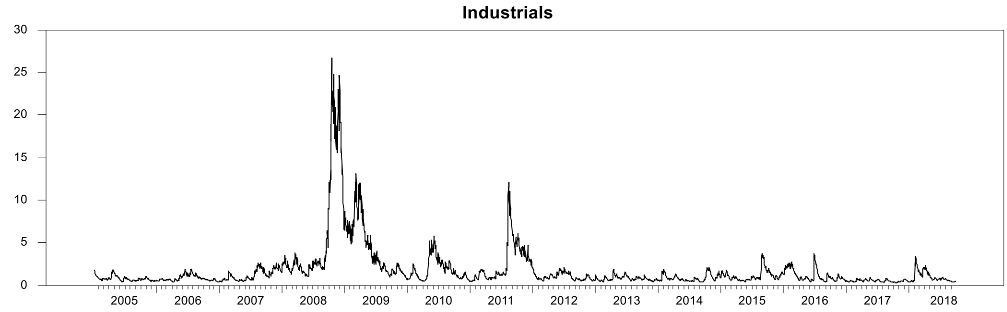

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